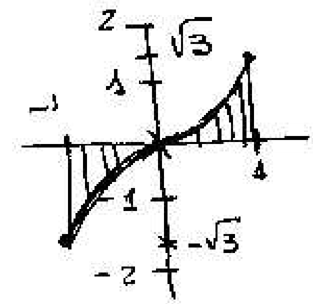
Integral definida - Soluciones -

(46)

$$\int_{-1}^{1} f(x)\,dx = -\frac{1}{2}\int_{-1}^{1} -2x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int_{-1}^{1} \boxed{-2x}\,(4-x^2)^{1/2}\,dx =$$

$$= \left[-\frac{1}{\cancel{2}}\,\frac{(4-x^2)^{3/\cancel{2}}}{3/\cancel{2}}\right]_{-1}^{1} = \left[-\frac{\sqrt{(4-x^2)^3}}{3}\right]_{-1}^{1} = \left(-\frac{\sqrt{4-1}}{3}\right) - \left(-\frac{\sqrt{4-1}}{3}\right) = 0.$$



$$I = \int_{0}^{1} f(x)\,dx = -\frac{1}{2}\int_{0}^{1} \boxed{-2x}\,\sqrt{4x^2} =$$

$$= -\frac{1}{2}\int_{0}^{1} \boxed{-2x}\,(4-x^2)^{1/2} = \left[-\frac{1}{\cancel{2}}\,\frac{(4-x^2)^{3/\cancel{2}}}{3/\cancel{2}}\right]_{0}^{1}$$

$$= \left[-\frac{1}{3}\sqrt{(4-x^2)^3}\right]_{0}^{1} = -\frac{1}{3}\sqrt{3^3} - \left(-\frac{1}{3}\,2^3\right) = -\frac{1}{\cancel{3}}\cancel{3}\sqrt{3} + \frac{8}{3} = \frac{8}{3} - \sqrt{3}$$

$$A = 2I = 2\left(\frac{8}{3} - \sqrt{3}\right) = \boxed{\frac{16}{3} - 2\sqrt{3}} \text{ u.s.}$$

(45)

$$f(x) = \begin{cases} (x+2)^2 - 4 & \text{si } x < 0 \\ -(x-2)^2 + 4 & \text{si } x \geq 0 \end{cases}$$

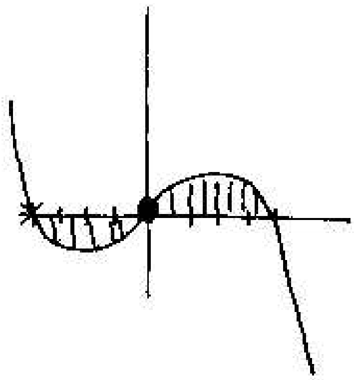
$\boxed{\text{P. corte con el eje } x}$

$$x^2 + 4x + \cancel{4} - \cancel{4} = 0 \Rightarrow x(x+4) = 0 \begin{cases} \cancel{x=0} \\ x = -4 \end{cases}$$

$$-(x^2 - 4x + 4) + 4 = 0 \Rightarrow -x^2 + 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow \boxed{\begin{matrix} x = 0 \\ x = 4 \end{matrix}}$$

$$I_1 = \int_{-1}^{0} (x+2)^2 - 4\,dx =$$

$$\left[\frac{(x+2)^3}{3} - 4x\right]_{-1}^{0} = \frac{8}{3} - \left(\frac{1}{3} + 4\right) = \frac{7}{3} - 4 = -\frac{5}{3}$$



$$I_2 = \int_{0}^{1} -(x-2)^2 + 4 = \left[-\frac{(x-2)^3}{3} + 4x\right]_{0}^{1} = \left[-\frac{(-1)}{3} + 4\right] -$$

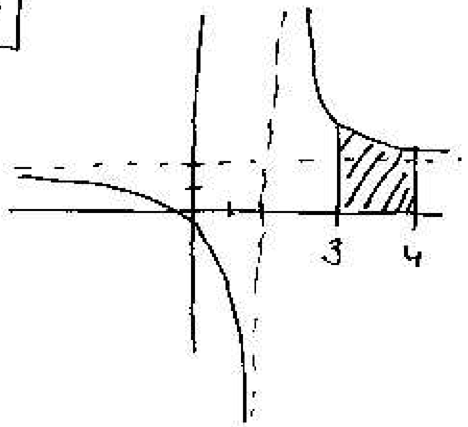
$$\left[-\frac{-8}{3} + \cancel{0}\right] = \frac{1}{3} + 4 - \left(\frac{8}{3} + \cancel{0}\right) = \frac{13}{3} - \frac{8}{3} = \frac{5}{3}$$

$$A = |I_1| + |I_2| = \frac{5}{3} + \frac{5}{3} = \boxed{\frac{10}{3}} \text{ u.s.}$$

(44) A. Horizontal $\rightarrow \lim_{x \to \infty} \frac{2x+1}{x-2} = 2 \Rightarrow \boxed{y=2}$

A. vertical $\lim_{x \to 2^-} \frac{2x+1}{x-2} = \frac{5}{0^-} = -\infty$

$\lim_{x \to 2^+} \frac{2x+1}{x-2} = \frac{5}{0^+} = +\infty.$



$$A = \int_3^4 \frac{2x+1}{x-2}\,dx = \int_3^4 2 + \frac{5}{x-2}\,dx$$

$$\begin{array}{r|l} \cancel{2x}+1 & x-2 \\ -\cancel{2x}+4 & 2 \\ \hline 5 & \end{array}$$

$$= \Big[2x + 5\ln|x-2|\Big]_3^4 =$$

$$= (2\cdot 4 + 5\cdot\ln 2) - (2\cdot 3 + 5\cdot\ln(3-2)) = 8 + 5\ln 2 - 6 - 5\ln 1 =$$

$$= \boxed{2 + 5\ln 2}$$

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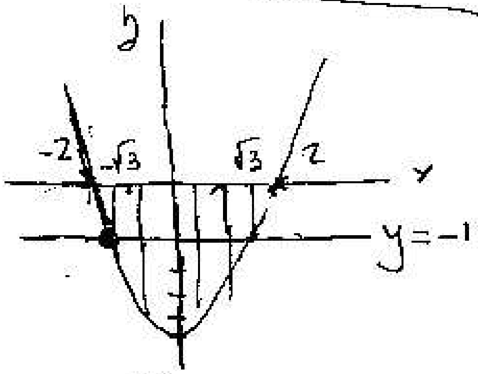
(43) $y = x^2 - 4$

$y = -1$



P. corte de las dos curvas

$$\left.\begin{array}{l} y = x^2 - 4 \\ y = -1 \end{array}\right\} \begin{array}{l} x^2 - 4 = -1 \\ x^2 = 3 \Rightarrow x = \pm\sqrt{3} \end{array}$$

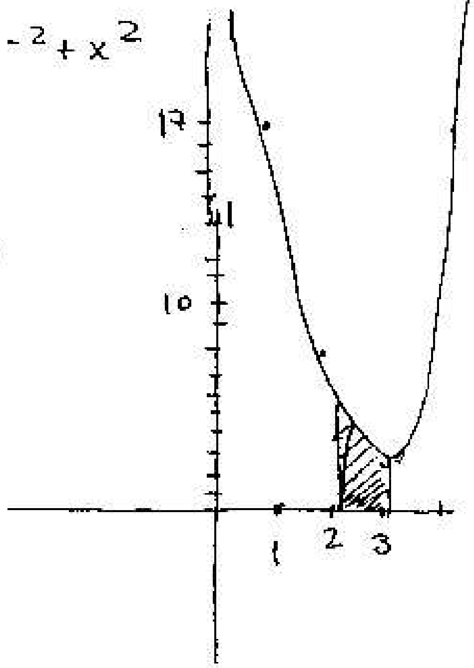
$$A = \int_{-\sqrt{3}}^{\sqrt{3}} f(x) - g(x)\,dx = \int_{-\sqrt{3}}^{\sqrt{3}} x^2 - 4 - (-1)\,dx = \int_{-\sqrt{3}}^{\sqrt{3}} x^2 - 3\,dx =$$

$$= \left[\frac{x^3}{3} - 3x\right]_{-\sqrt{3}}^{\sqrt{3}} = \left[\frac{\sqrt{3}^3}{3} - 3\cdot\sqrt{3}\right] - \left[-\frac{\sqrt{3}^3}{3} - 3\cdot(-\sqrt{3})\right] =$$

$$= \left(\frac{\cancel{3}\sqrt{3}}{\cancel{3}} - 3\sqrt{3}\right) - \left(-\frac{\cancel{3}\sqrt{3}}{\cancel{3}} + 3\sqrt{3}\right) = \sqrt{3} - 3\sqrt{3} + \sqrt{3} - 3\sqrt{3} = -4\sqrt{3}$$

$$A = |-4\sqrt{3}| = 4\sqrt{3} \text{ u.s.}$$

(4 2) $f(x)=\frac{a}{x^2}+x^2 \quad x>0. \qquad f(x)=a\cdot x^{-2}+x^2$



$f'(x)=a\cdot(-2)x^{-3}+2x=\frac{-2a}{x^3}+2x$

$f'(1)=\frac{-2a}{1}+2=0 \Rightarrow 2a=2 \Rightarrow \boxed{a=1}$

b) $f(x)=\frac{16}{x^2}+x^2$

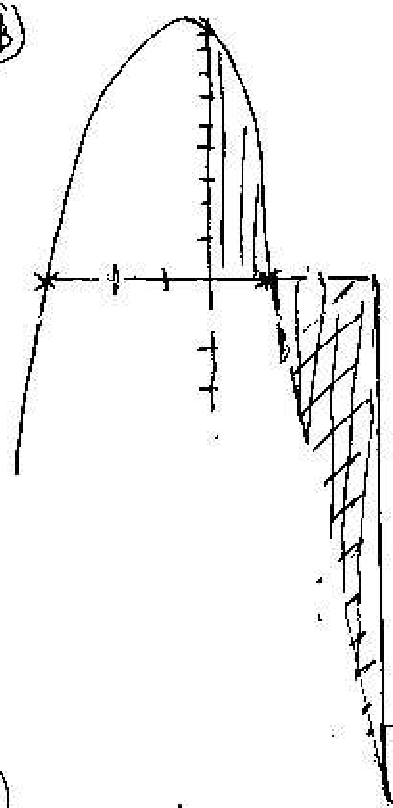
$\lim_{x\to 0^+}\frac{16}{x^2}+x^2=\frac{16}{0^+}+0=+\infty$

x	0^+	1	2	3	4	$+\infty$
y	$+\infty$	17	8	$\frac{25}{9}$	17	$+\infty$

$A=\int_2^3 16x^{-2}+x^2=\left[\frac{16x^{-1}}{-1}+\frac{x^3}{3}\right]_2^3=\left[-\frac{16}{x}+\frac{x^3}{3}\right]_2^3=\left(-\frac{16}{3}+\frac{27}{3}\right)-\left(\frac{-16}{2}+\frac{8}{3}\right)$

$=\left(-\frac{16}{3}+9\right)-\left(-8+\frac{8}{3}\right)=-\frac{16}{3}+9+8-\frac{8}{3}=\cancel{-8}+9+\cancel{8}=9$ u.s.

(4B)



$y=-3x^2-6x+9.$

P. corte con el eje X

$-3x^2-6x+9=0 \Rightarrow x^2+2x-3=0$

$x=\frac{-2\pm\sqrt{4+12}}{2}=\frac{-2\pm 4}{2} \quad \frac{2}{2}=1 \quad \frac{-6}{2}=-3$

PRIMITIVA DE $f(x)$

$F(x)=\int -3x^2-6x+9=-\frac{3x^3}{3}-\frac{6x^2}{2}+9x+k$

$=-x^3-3x^2+9x+k.$

$F(1)=-1-3+9+k$

$10=-1-3+9+k$

$10+1+3-9=k \Rightarrow \boxed{k=5}$

$\boxed{F(x)=-x^3-3x^2+9x+5}$

(b)

$I_1=\int_0^1 -3x^2-6x+9=\left[-x^3-3x^2+9x\right]_0^1=1-3+9=7$

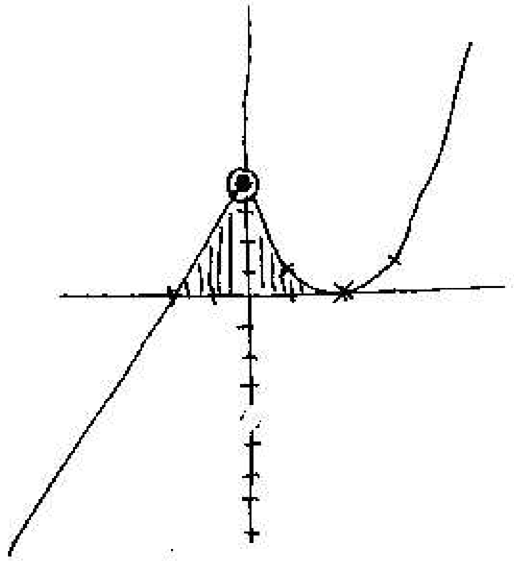
$I_2=\int_1^3 -3x^2-6x+9=\left[-x^3-3x^2+9x\right]_1^3=-27-\cancel{27}+\cancel{27}-7=-34.$

$A=|I_1|+|I_2|=7+34=41$ u.s.

(40) $f(x)=\begin{cases} 2x+4 & \text{si } x\leq 0 \\ (x-2)^2 & \text{si } x>0. \end{cases}$

x	-2	-1	0
$2x+4$	0	2	4

x	0^+	1	2	3
$(x-2)^2$	4	1	0	1



$I_1=\int_{-2}^{0} 2x+4 = \left[\frac{\cancel{2}x^2}{\cancel{2}}+4x\right]_{-2}^{0} =$

$0-[4-8]=4$,

$I_2=\int_0^2 (x-2)^2\,dx = \left[\frac{(x-2)^3}{3}\right]_0^2 =$

$=\frac{8}{3}$

$A=|I_1|+|I_2|=4+\frac{8}{3}=\frac{20}{3}$ u.s.

(39) $f(x)=4e^{4x}+a$

a) $F_1(x)=4e^{4x}+ax$ $\quad F_1'(x)=4\cdot e^{4x}\cdot 4+a=16e^{4x}+a\neq f(x)$

No es primitiva

$F_2(x)=e^{4x}+ax$ $\quad F_2'(x)=e^{4x}\cdot 4+a=f(x)$

F_2 es primitiva de $f(x)$

b) $\int_0^1 f(x)\,dx=e^4$

$\int_0^1 4e^{4x}+a=\left[e^{4x}+ax\right]_0^1= e^4+a-(e^0-0)=\cancel{e^4}+a-1=\cancel{e^4}$ $\boxed{a=1}$

(38) $f(x)=x+\frac{a}{x^3}$

a) $F(x)=\int x+\frac{a}{x^3}\,dx=\int x+a\cdot x^{-3}\,dx=\frac{x^2}{2}+\frac{a\,x^{-2}}{-2}+K=\frac{x^2}{2}-\frac{a}{2x^2}+K.$

b) $G(x)=F'(x)+2 \Rightarrow G'(x)=f(x)+2$. No es primitiva de $f(x)$.

c) $\int_1^2 f(x)\,dx = 1,5 \Rightarrow \int_1^2 x + \frac{a}{x^3}\,dx = \int_1^2 x + a x^{-3}\,dx =$

$= \left[\frac{x^2}{2} + \frac{a x^{-2}}{-2}\right]_1^2 = \left[\frac{x^2}{2} - \frac{a}{2x^2}\right]_1^2 = \left(\frac{4}{2} - \frac{a}{8}\right) - \left(\frac{1}{2} - \frac{a}{2}\right) =$

$= 2 - \frac{a}{8} - \frac{1}{2} + \frac{a}{2} = 1,5 \Rightarrow 16 - a - 4 + 4a = 12$

$3a = 0 \Rightarrow \boxed{a = 0}$

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(37) $f(x) = x^3 - 27 + a x e^{x^2}$

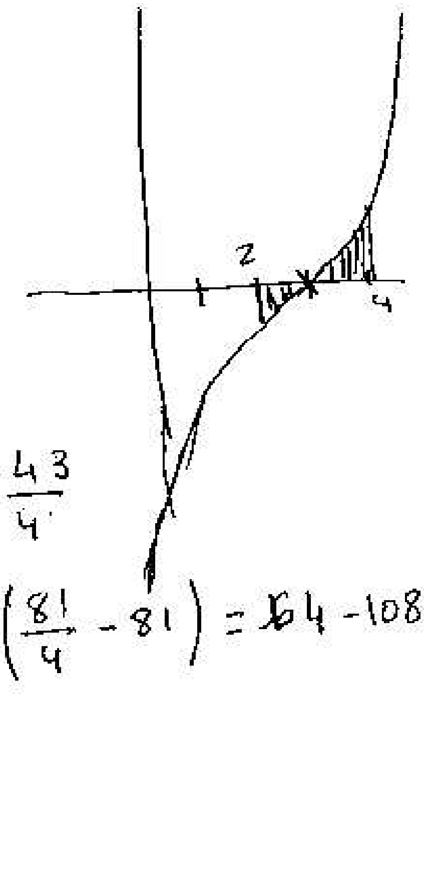
a) $F(x) = \int x^3 - 27 + a x e^{x^2} = \int x^3 dx - \int 27 dx + a \int x e^{x^2} dx =$

$= \frac{x^4}{4} - 27x + a \frac{1}{2} \int \boxed{2} x\, e^{x^2} dx = \frac{x^4}{4} - 27x + \frac{a}{2} e^{x^2} + k.$

b) $a = 0 \quad f(x) = x^3 - 27$

P. corte con el eje X $\rightarrow x^3 - 27 = 0 \Rightarrow x^3 = 27 \Rightarrow x = \sqrt[3]{27} = 3$

x	-1	1	3	4
y	-28	-26	0	37



$I_1 = \int_2^3 x^3 - 27 = \left[\frac{x^4}{4} - 27x\right]_2^3 =$

$\left(\frac{81}{4} - 81\right) - \left(\frac{16}{4} - 54\right) =$

$\frac{81}{4} - 81 - \frac{16}{4} + 54 = \frac{65}{4} - 27 = \frac{65 - 108}{4} = -\frac{43}{4}$

$I_2 = \int_3^4 x^3 - 27 = \left[\frac{x^4}{4} - 27x\right]_3^4 = \left(\frac{256}{4} - 108\right) - \left(\frac{81}{4} - 81\right) = 64 - 108 - \frac{81}{4} + 81$

$37 - \frac{81}{4} = \frac{148 - 81}{4} = \frac{67}{4}$

$A = \left|-\frac{43}{4}\right| + \frac{67}{4} = \frac{110}{4} = \frac{55}{2}$ u.s.

(36) $f(x) = a e^{x/3} + \frac{1}{x^2} \quad (x>0)$

a) $\int_1^2 f(x)\,dx = \int_1^2 a e^{x/3} + \frac{1}{x^2}\,dx = a\int_1^2 e^{x/3}\,dx + \int_1^2 x^{-2}\,dx =$

$= a\cdot 3\int_1^2 \boxed{\frac{1}{3}} e^{x/3} + \left[\frac{x^{-2+1}}{-2+1}\right]_1^2 = 3a\left[e^{x/3}\right]_1^2 + \left[\frac{x^{-1}}{-1}\right]_1^2 =$

$= 3a e^{2/3} - 3a e^{1/3} + \left[-\frac{1}{x}\right]_1^2 = 3a e^{2/3} - 3a e^{1/3} - \frac{1}{2} + 1 =$

$= 3a e^{2/3} - 3a e^{1/3} + \frac{1}{2}$

b) $F(x) = 3a e^{x/3} - \frac{1}{x} + k$ (calculada en a)

$F(1) = 3a e^{1/3} - 1 + k = 0 \Rightarrow k = 1 + 3a e^{1/3}$

$F(2) = 3a e^{2/3} - \frac{1}{2} + k = \frac{1}{2} \Rightarrow 3a e^{2/3} - \cancel{\frac{1}{2}} + \cancel{1} + 3a e^{1/3} = \cancel{\frac{1}{2}}$

$3a e^{2/3} + 3a e^{1/3} = 0$

$a\left(3e^{2/3} + 3e^{1/3}\right) = 0 \Rightarrow a = \frac{0}{(3e^{2/3} + 3e^{1/3})} = 0 \Rightarrow \boxed{a=0}$

$k = 1 + 3\cdot 0\, e^{1/3} \Rightarrow \boxed{k=1}$

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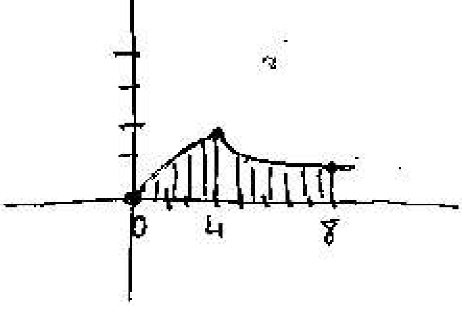
(35) $f(x) = \begin{cases} \sqrt{x} & \text{si } x < 4 \\ \frac{x-2}{x-3} & \text{si } x \geq 4. \end{cases}$

x	0	1	4
$\sqrt{x}$	0	1	2

$x\to$	4	5	6	$+\infty$
$\frac{x-2}{x-3}$	2	$\frac{3}{2}$	$\frac{4}{3}$	1



$\lim_{x\to+\infty} \frac{x-2}{x-3} = 1$ A. horiz. $y=1$

$I_1 = \int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}\,dx = \left[\frac{x^{3/2}}{3/2}\right]_0^4 = \left[\frac{2\sqrt{x^3}}{3}\right]_0^4 = \frac{2\sqrt{64}}{3} = \frac{16}{3}$

$I_2 = \int_4^8 \frac{x-2}{x-3}\,dx = \int_4^8 1 + \frac{1}{x-3}\,dx = \left[x + \ln|x-3|\right]_4^8 =$

$= 8 + \ln 5 - (4 + \ln 1) = 4 + \ln 5$

$\boxed{A} = \frac{16}{3} + 4 + \ln 5 = \boxed{\frac{28}{3} + \ln 5}$

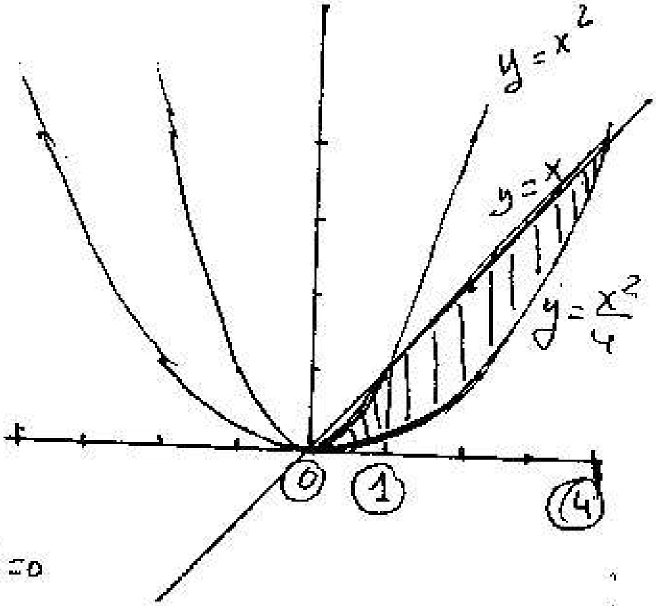
$x-2 \;|\; x-3$
$-x+3 \quad 1$
1

(34) $y = x$, $y = x^2$, $y = x^2/4$

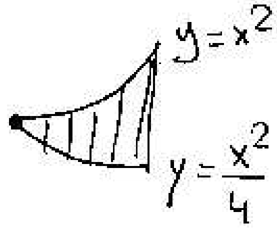
x	0	2
y	0	2

x	-2	0	2
y	4	0	4

x	-4	-2	0	2	4
y	4	1	0	1	4



P_1



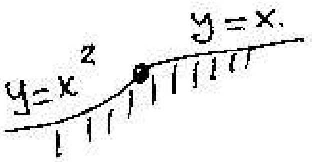
P. corte

$x^2 = \frac{x^2}{4}$

$4x^2 = x^2 \Rightarrow$

$3x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

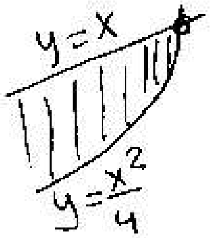
P_2



P. corte

$\left.\begin{array}{l} y = x^2 \\ y = x \end{array}\right\}$ $x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \begin{cases} x = 0 \\ x = 1 \end{cases}$

P_3



P. corte

$\left.\begin{array}{l} y = x \\ y = \frac{x^2}{4} \end{array}\right\}$ $x = \frac{x^2}{4} \Rightarrow 4x = x^2 \Rightarrow x^2 - 4x = 0 \Rightarrow$

$x(x-4) = 0 \begin{cases} x = 0 \\ x = 4 \end{cases}$

$$I_1 = \int_0^1 x^2 - \frac{x^2}{4} = \int_0^1 \frac{3x^2}{4} = \frac{3}{4}\int_0^1 x^2 = \left[\frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}}\right]_0^1 = \frac{1}{4}$$

$$I_2 = \int_1^4 x - \frac{x^2}{4} = \left[\frac{x^2}{2} - \frac{1}{4}\frac{x^3}{3}\right]_1^4 = \left(\frac{16}{2} - \frac{4^3}{4\cdot 3}\right) - \left(\frac{1}{2} - \frac{1}{12}\right) =$$

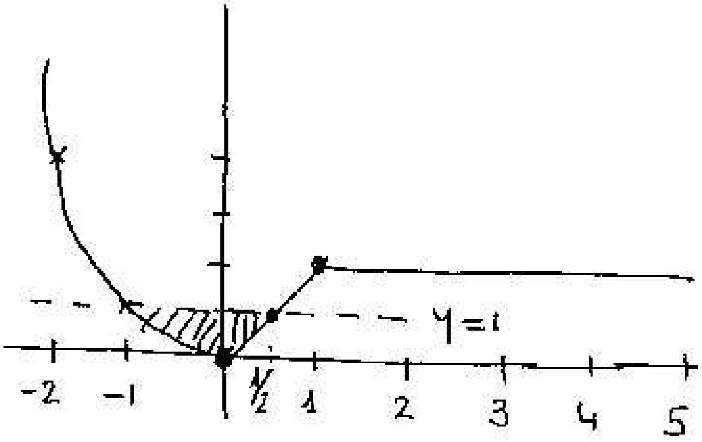
$$= 8 - \frac{16}{3} - \frac{1}{2} + \frac{1}{12} = \frac{96 - 64 - 6 + 1}{12} = \frac{17}{12}$$

$$A = \frac{1}{4} + \frac{17}{12} = \frac{3+17}{12} = \frac{20}{12} = \boxed{\frac{5}{3}} \text{ u.S.}$$

(33) $f(x) = \begin{cases} x^2 & \text{si } x < 0 \\ 2x & 0 \le x < 1 \\ 2 & x \ge 1 \end{cases}$

x	-2	-1	0^-
x^2	4	1	0

x	0	1^-
$2x$	0	2



P. corte de $f(x)$ con $y=1$

(*) $x^2 = 1 \Rightarrow x = \pm 1$

$x = 1$ no vale porque $1 > 0$.

(**) $2x = 1 \Rightarrow x = 1/2$

(*) $2 = 1$ no corta

$$I_1 = \int_{-1}^{0} x^2 - 1 = \left[\frac{x^3}{3} - x\right]_{-1}^{0} = 0 - \left(\frac{-1}{3} + 1\right) = -\frac{2}{3}$$

$$I_2 = \int_0^{1/2} 2x - 1 = \left[\frac{2x^2}{2} - x\right]_0^{1/2} = \left[x^2 - x\right]_0^{1/2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$A = |I_1| + |I_2| = \frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12}$$

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(32) $f(x) = x^2 - 6x + 8$

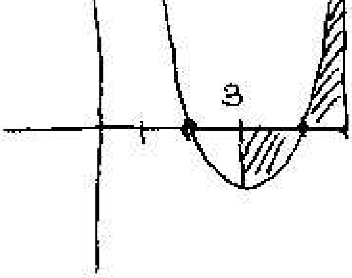
a) $F(x) = \int x^2 - 6x + 8 = \frac{x^3}{3} + \frac{6x^2}{2} + 8x + k$

$F(3) = 10 \Rightarrow \frac{27}{3} + \frac{54}{2} + \quad + k = 10 \Rightarrow 9 + 27 + 24 + k = 10$

$k = 10 - 9 - 27 - 24 = -50 \Rightarrow \boxed{F(x) = \frac{x^3}{3} + 3x^2 + 8x - 50}$

(b) P. corte con el eje x

$$\left.\begin{array}{l} y = x^2 - 6x + 8 \\ y = 0 \end{array}\right\} \begin{array}{l} 0 = x^2 - 6x + 8 \\ x = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm 2}{2} \begin{array}{l} 4 \\ 2 \end{array} \end{array}$$



$$I_1 = \int_3^4 x^2-6x+8 = \left[\frac{x^3}{3} - \frac{6x^2}{2} + 8x\right]_3^4 = \left(\frac{64}{3} - 48 + 32\right) - \left(\frac{27}{3} - 27 + 24\right)$$

$$= \frac{64}{3} - 48 + 32 - \frac{27}{3} + 27 - 24 = \frac{37}{3} - 13 = \frac{37-39}{3} = \frac{-2}{3}$$

$$I_2 = \int_4^5 x^2-6x+8 = \left[\frac{x^3}{3} - 3x^2 + 8x\right]_4^5 = \left(\frac{125}{3} - 75 + 40\right) - \left(\frac{64}{3} - 48 + 32\right)$$

$$= \frac{125}{3} - 35 - \frac{64}{3} + 16 = \frac{61}{3} - 19 = \frac{61-57}{3} = \frac{4}{3}$$

$$A = |I_1| + |I_2| = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ u.s.}$$

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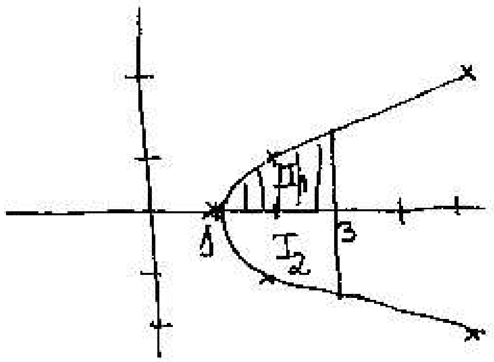
(31) $x = y^2+1 \Rightarrow x-1 = y^2 \begin{cases} y = \sqrt{x-1} \\ y = -\sqrt{x-1} \end{cases}$

x	1	2	5
$\sqrt{x-1}$	0	1	2

x	1	2	5
$-\sqrt{x-1}$	0	-1	-2



$$I_1 = \int_1^3 \sqrt{x-1} = \int_1^3 (x-1)^{1/2} dx =$$

$$= \left[\frac{(x-1)^{3/2}}{3/2}\right]_1^3 = \left[\frac{2\sqrt{(x-1)^3}}{3}\right]_1^3 = \frac{2\sqrt{8}}{3} - 0$$

$I_1 = I_2$ porque el recinto es simétrico.

$$A = 2I_1 = \frac{2\cdot\sqrt{8}}{3}\cdot 2 = \frac{4\sqrt{8}}{3} = \frac{4\sqrt{2^3}}{3} = \boxed{\frac{8\sqrt{2}}{3}} \text{ u.s.}$$

---

(30) P. corte de ambas curvas

$$\begin{cases} y = x^2-4x \\ y = -x^3 \end{cases} \quad \begin{aligned} &-x^3 = x^2-4x \\ &x^3+x^2-4x = 0 \Rightarrow x(x^2+x-4) = 0 \end{aligned} \begin{cases} x = 0 \\ x = \frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} \begin{cases} 1 \\ -2 \end{cases} \end{cases}$$

$$I_1 = \int_{-2}^0 x^2-4x+x^3 = \left[\frac{x^3}{3} - \frac{4x^2}{2} + \frac{x^4}{4}\right]_{-2}^0 = 0 - \left(\frac{-8}{3} - 8 + 4\right) = \frac{32}{3}$$

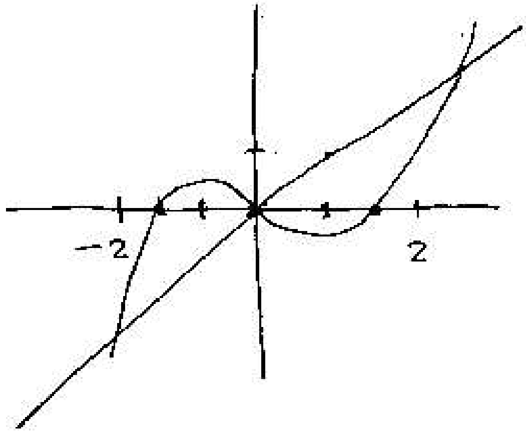
$$I_2 = \int_0^1 x^2-4x+x^3 = \left[\frac{x^3}{3} - 2x^2 + \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - 2 + \frac{1}{4} = \frac{4-24+3}{12} = \frac{-17}{12}$$

$$A = \frac{32}{3} + \frac{17}{12} = \frac{145}{12} \text{ u.s.}$$

(29) P. corte con el eje X de $y = x^3 - 3x$

$x^3 - 3x = 0$

$x(x^2-3) = 0 \begin{cases} x = 0 \\ x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3} \end{cases}$



b) $\boxed{\text{Área}}$

$\boxed{\text{P. corte de ambas curvas}}$

$\left.\begin{array}{l} y = x^3 - 3x \\ y = x \end{array}\right\} x^3 - 3x = x \Rightarrow x^3 - 4x = 0 \Rightarrow x(x^2-4) = 0 \begin{cases} x = 0 \\ x = \pm 2 \end{cases}$

$I_1 = \int_{-2}^{0} x^3 - 3x - x = \int_{-2}^{0} x^3 - 4x = \left[\frac{x^4}{4} - \frac{4x^2}{2}\right]_{-2}^{0} = \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 0 - (4-8) = 4$

$I_2 = \int_0^2 x^3 - 3x - x = \int_0^2 x^3 - 4x = \left[\frac{x^4}{4} - 2x^2\right]_0^2 = 4 - 8 = -4$

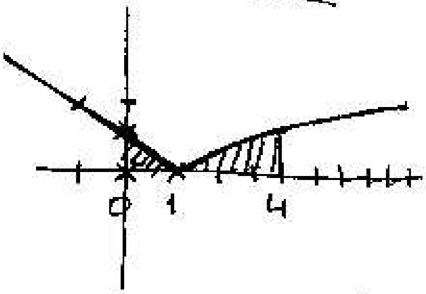
$A = |I_1| + |I_2| = 4 + 4 = \boxed{8}$

---

(28) $f(x) = \begin{cases} 1 - x & \text{si } x < 1 \\ \sqrt{x} - \frac{1}{x} & \text{si } x \geq 1 \end{cases}$

x	-1	0	1^-
y	2	1	0

x	1	4	9
$y = \sqrt{x} - \frac{1}{x}$	0	$\frac{7}{4}$	$\frac{26}{9}$



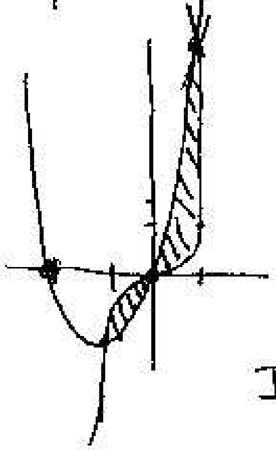
$I_1 = \int_0^1 1 - x \, dx = \left[x - \frac{x^2}{2}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$

$I_2 = \int_1^4 \sqrt{x} - \frac{1}{x} \, dx = \int_1^4 x^{1/2} - \frac{1}{x} = \left[\frac{x^{3/2}}{3/2} - \ln|x|\right]_1^4 = \left[\frac{2\sqrt{x^3}}{3} - \ln|x|\right]_1^4$

$= \frac{2 \cdot \sqrt{64}}{3} - \ln 4 - \left(\frac{2}{3} - \ln 1\right) = \frac{16}{3} - \ln 4 - \frac{2}{3} = \frac{14}{3} - \ln 4$

$A = |I_1| + |I_2| = \frac{1}{2} + \frac{14}{3} - \ln 4 = \frac{31}{6} - \ln 4$ u.s.

(27) $y = x^2 + 2x$



Puntos de corte de ambas curvas

$$\left.\begin{array}{l} y = x^2+2x \\ y = x^3 \end{array}\right\} \quad x^3 = x^2+2x \Rightarrow x^3-x^2-2x=0$$

$$x(x^2-x-2)=0 \begin{cases} \boxed{x=0} \\ x^2-x-2=0 \end{cases}$$

$$x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{cases} \boxed{2} \\ \boxed{-1} \end{cases}$$

$$I_1 = \int_{-1}^{0} x^2+2x-x^3 = \left[\frac{x^3}{3} + \frac{2x^2}{2} - \frac{x^4}{4}\right]_{-1}^{0} = -\left(\frac{-1}{3}+1-\frac{1}{4}\right) =$$

$$\frac{1}{3} - 1 + \frac{1}{4} = \frac{4-12+3}{12} = \frac{-5}{12}$$

$$I_2 = \int_0^2 x^2+2x-x^3 = \left[\frac{x^3}{3} + x^2 - \frac{x^4}{4}\right]_0^2 = \frac{8}{3} + \cancel{4} - \cancel{\frac{16}{4}} = \frac{8}{3}$$

$$A = |I_1| + |I_2| = \frac{5}{12} + \frac{8}{3} = \frac{5+32}{12} = \frac{37}{12} \text{ u.s.}$$

---

(26) $f(x) = 1 - \frac{1}{x^2}$

a) $F(x) = \int 1 - \frac{1}{x^2}\,dx = \int 1 - x^{-2}\,dx = x - \frac{x^{-1}}{-1} + k = x + \frac{1}{x} + k.$

$F(1) = 1+1+k = 3 \Rightarrow k = 1$

$F(x) = x + \frac{1}{x} + 1$

b) Puntos de corte de la curva con el eje X

$$\left.\begin{array}{l} y = 1 - \frac{1}{x^2} \\ y = 0 \end{array}\right\} \quad 0 = 1 - \frac{1}{x^2} \Rightarrow 1 = \frac{1}{x^2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Entre $x = \frac{1}{2}$ y $x = 2$ hay un punto de corte en $x = 1$

$$I_1 = \int_{1/2}^{1} 1 - \frac{1}{x^2} = \int_{1/2}^{1} 1 - x^{-2}\,dx = \left[x + \frac{1}{x}\right]_{1/2}^{1} = 1+1-\left(\frac{1}{2} + \frac{1}{1/2}\right) = \cancel{2} - \frac{1}{2} - \cancel{2} = -\frac{1}{2}$$

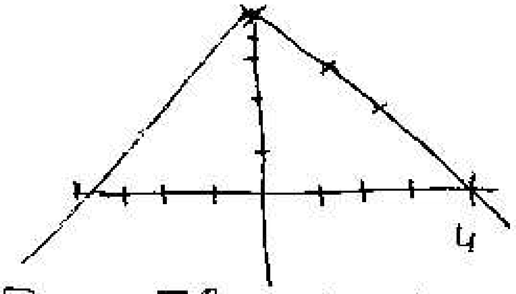
$$I_2 = \int_1^2 1 - \frac{1}{x^2} = \int_1^2 1 - x^{-2}\,dx = \left[x + \frac{1}{x}\right]_1^2 = 2 + \frac{1}{2} - (1+1) = \frac{1}{2}$$

$$A = |I_1| + |I_2| = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1 \text{ u.s.}$$

(25) $f(x) = \begin{cases} 4+x & \text{si } x<0 \\ 4-x & \text{si } x \geq 0 \end{cases}$

x	-2	-1	0^-
$4+x$	2	3	4

x	0	1	2
$4-x$	4	3	2



P. corte $f(x)$ con el eje x

$4+x=0 \Rightarrow x=-4$

$4-x=0 \Rightarrow x=4$

$$I_1 = \int_{-4}^{0} 4+x \, dx = \left[4x+\frac{x^2}{2}\right]_{-4}^{0} = -(-16+8) = 8$$

$$I_2 = \int_{0}^{4} 4-x \, dx = \left[4x-\frac{x^2}{2}\right]_{0}^{4} = 16-8=8$$

$\boxed{A = 8+8 = 16 \text{ u.s}}$

---

(24) Puntos de corte de las curvas.

$$\left.\begin{array}{l} y=x^2-4x \\ y=-x^2+4x \end{array}\right\} \begin{array}{l} x^2-4x=-x^2+4x \\ 2x^2-8x=0 \Rightarrow x(2x-8)=0 \end{array} \begin{cases} x=0 \\ 2x-8=0 \Rightarrow x=4. \end{cases}$$

$$\int_0^4 x^2-4x-(-x^2+4x)\,dx = \int_0^4 2x^2-8x \, dx = \left[\frac{2x^3}{3}-\frac{8x^2}{2}\right]_0^4 =$$

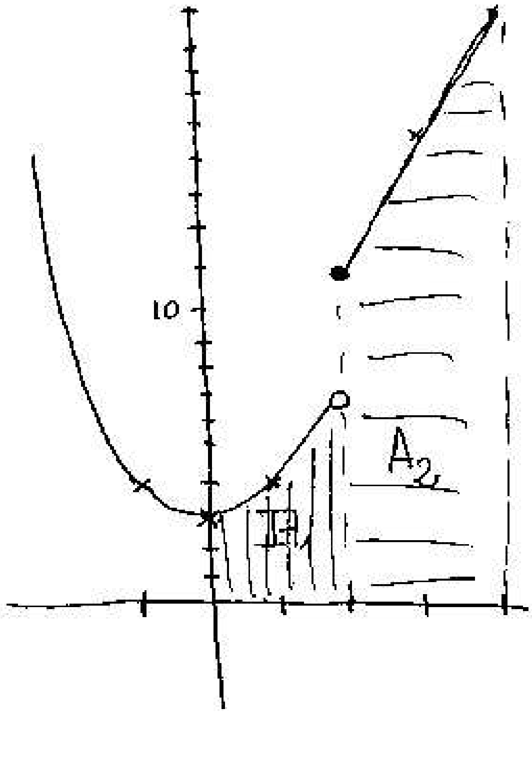
$$\frac{128}{3}-\frac{128}{2} = -\frac{128}{6} = -\frac{64}{3} \qquad A = \left|-\frac{64}{3}\right| = \frac{64}{3} \text{ u.s.}$$

---

(23) $f(x) = \begin{cases} x^2+3 & \text{si } x<2 \\ 4x+3 & \text{si } x \geq 2 \end{cases}$

x	-1	0	1	2^-
x^2+3	4	3	4	7

x	2	3	4
$4x+3$	11	15	19



$$I_1 = \int_0^2 x^2+3 = \left[\frac{x^3}{3}+3x\right]_0^2 = \frac{8}{3}+6 = \frac{26}{3}$$

$$I_2 = \int_2^4 4x+3 = \left[\frac{4x^2}{2}+3x\right]_2^4 =$$

$$= \left[2x^2+3x\right]_2^4 = 32+12-(8+6) = 30$$

$$A = |I_1|+|I_2| = \frac{26}{3}+30 = \boxed{\frac{116}{3}} \text{ u.s}$$

(22) Puntos de corte de las dos funciones

$$\left.\begin{array}{l} y = x^2+3 \\ y = 4x \end{array}\right\} x^2+3 = 4x \Rightarrow x^2-4x+3 = 0 \Rightarrow$$

$$x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} \left\langle \begin{array}{l} 3 \\ 1 \end{array}\right.$$

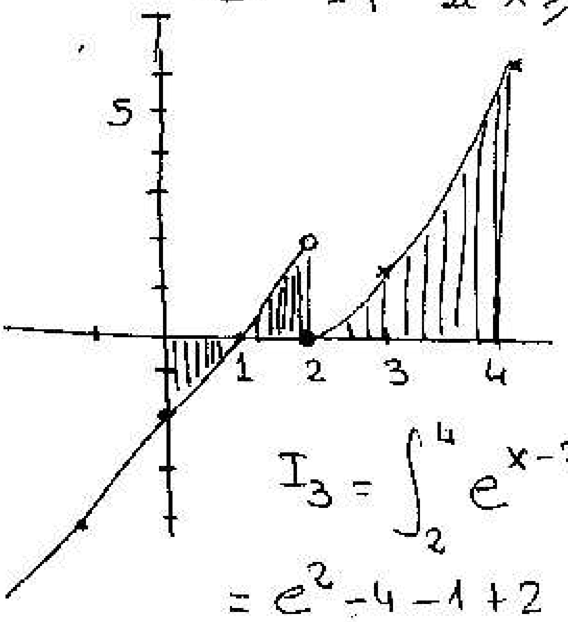
$$I = \int_1^3 x^2+3-4x = \left[\frac{x^3}{3} + 3x - \frac{4x^2}{2}\right]_1^3 = \left(\cancel{\frac{27}{3}} + \cancel{9} - \cancel{18}\right) - \left(\frac{1}{3}+3-2\right)$$

$$= -\left(\frac{1}{3}+1\right) = -\frac{4}{3} \qquad A = |I| = \frac{4}{3} \text{ u.s.}$$

(21) $f(x) = \begin{cases} 2x-2 & \text{si } x < 2 \\ e^{x-2}-1 & \text{si } x \geq 2 \end{cases}$

x	-1	1	2^-
$2x-2$	-4	0	2

x	2	3	4
$e^{x-2}-1$	0	$e-1$	e^2-1
		$\simeq 1'7$	$\simeq 6'3$



$$I_1 = \int_0^1 2x-2 = \left[\frac{2x^2}{2} - 2x\right]_0^1 = \left[x^2-2x\right]_0^1 = -1$$

$$I_2 = \int_1^2 2x^2-2 = \left[x^2-2x\right]_1^2 = (4-4)-(1-2) = 1$$

$$I_3 = \int_2^4 e^{x-2}-1 = \left[e^{x-2}-x\right]_2^4 = \left(e^2-4\right)-\left(e^0-2\right) =$$

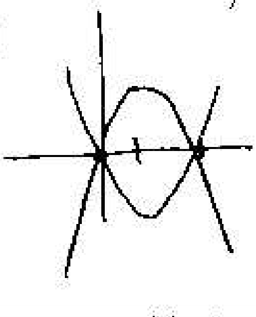
$$= e^2-4-1+2 = e^2-3$$

$$A = |I_1| + |I_2| + |I_3| = 1+1+e^2-3 = \boxed{e^2-1} \text{ u.s.}$$

(20) $f(x) = 2x-x^2$

a) $F(x) = \int 2x-x^2 = \frac{2x^2}{2} - \frac{x^3}{3} + k = x^2 - \frac{x^3}{3} + k.$ $\quad \boxed{F(x) = x^2 - \frac{x^3}{3} + 100}$

$F(3) = \cancel{3^2} - \cancel{\frac{3^3}{3}} + k = 100 \Rightarrow k = 100$

b) 

P. corte de las dos funciones

$$\left.\begin{array}{l} y = 2x-x^2 \\ y = x^2-2x \end{array}\right| \begin{array}{l} 2x-x^2 = x^2-2x \\ -2x^2+4x = 0 \Rightarrow x(-2x+4) = 0 \end{array} \left\langle \begin{array}{l} x = 0 \\ -2x+4 = 0 \Rightarrow x = 2 \end{array}\right.$$

$$\int_0^2 2x-x^2-(x^2-2x) = \int_0^2 -2x^2+4x = \left[-\frac{2x^3}{3} + \frac{4x^2}{2}\right]_0^2 = -\frac{16}{3} + 8 = \boxed{\frac{8}{3}} \text{ u.s}$$

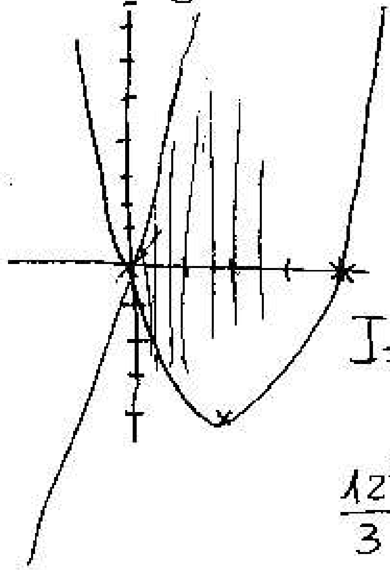
(19) $f(x) = x^2 - 4x$

a) $F(x) = \int x^2 - 4x \, dx = \frac{x^3}{3} - \frac{4x^2}{2} + k = \frac{x^3}{3} - 2x^2 + k$

$F(3) = \frac{27}{3} - 12 + k = 0 \Rightarrow 9 - 12 + k = 0 \Rightarrow k = 3$

$\boxed{F(x) = \frac{x^3}{3} - 2x^2 + 3}$

b)



P. corte de ambas curvas

$y = x^2 - 4x$
$y = 8x$

$8x = x^2 - 4x$

$x^2 - 12x = 0 \quad x(x-12) = 0 \quad x = 0, \; x = 12$

$I = \int_0^{12} x^2 - 4x - 8x = \int_0^{12} x^2 - 12x = \left[\frac{x^3}{3} - \frac{12x^2}{2}\right]_0^{12} =$

$\frac{12^3}{3} - \frac{12 \cdot 12^2}{2} = -\frac{12^3}{6} = -288$

$\boxed{A = |I| = 288 \text{ us}}$

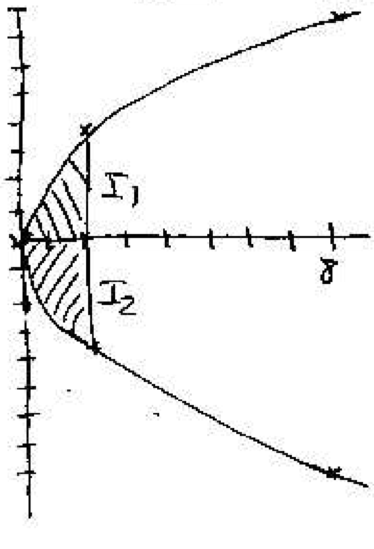
(18) $y^2 = 8x$

$y = \sqrt{8x}$

x	0	2	8
y	0	4	8

$y = -\sqrt{8x}$

x	0	2	8
y	0	-4	-8

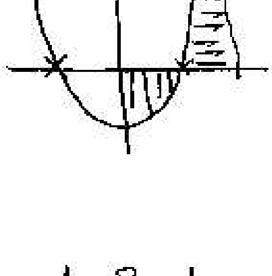


$I_1 = \int_0^2 \sqrt{8x} \, dx = \sqrt{8} \int_0^2 x^{1/2} dx =$

$\left[\sqrt{8} \cdot \frac{x^{3/2}}{3/2}\right]_0^2 = \sqrt{8}\left[\frac{2\sqrt{x^3}}{3}\right]_0^2 = \frac{\sqrt{8} \cdot 2 \cdot \sqrt{8}}{3} = \frac{16}{3}$

$I_1 = I_2 \Rightarrow A = 2I_1 = 2 \cdot \frac{16}{3} = \frac{32}{3}$ u.s.

(17) $f(x) = x^2 - 1$



a) $F(x) = \int x^2 - 1 \, dx = \frac{x^3}{3} - x + k$

$F(3) = \frac{27}{3} - 3 + k = 10 \Rightarrow 9 - 3 + k = 10 \Rightarrow k = 4$

$\boxed{F(x) = \frac{x^3}{3} - x + 4}$

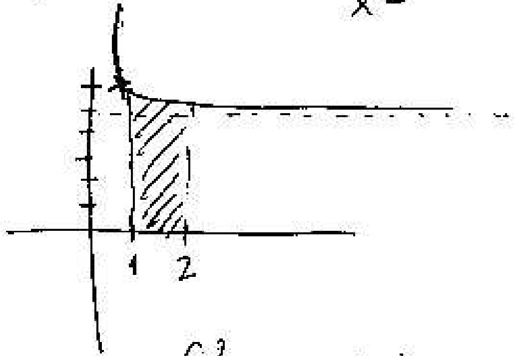
b) $I_1 = \int_0^1 x^2 - 1 = \left[\frac{x^3}{3} - x\right]_0^1 = \frac{1}{3} - 1 = -\frac{2}{3}$

$I_2 = \int_1^2 x^2 - 1 = \left[\frac{x^3}{3} - x\right]_1^2 = \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

$A = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = \boxed{2}$ u.s

(16) $f(x) = 5 + \frac{1}{x^2} \quad (x>0)$ $\qquad f(x) = 5 + x^{-2}$ $\qquad f'(x) = -2\,x^{-3} = \frac{-2}{x^3}$

$f'(2) = \frac{-2}{2^3} = -\frac{1}{4}$



b) $\lim_{x\to 0^+} 5 + \frac{1}{x^2} = 5 + \frac{1}{0^+} = +\infty$

x	0^+	1	2	3	$+\infty$
y	$+\infty$	6	$5+\frac{1}{4}$	$5+\frac{1}{9}$	5

$\lim_{x\to +\infty} 5 + \frac{1}{x^2} = 5 + \frac{1}{\infty} = 5$

$I = \int_1^2 5 + \frac{1}{x^2}\,dx = \int_1^2 5 + x^{-2}\,dx$

$= \left[5x + \frac{x^{-1}}{-1}\right]_1^2 = \left[5x - \frac{1}{x}\right]_1^2 =$

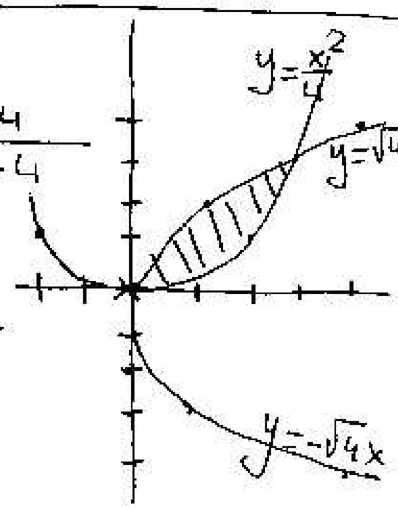
$= 10 - \frac{1}{2} - (5-1) = 6 - \frac{1}{2} = \boxed{\frac{11}{2}}$ u.s.

---

(15) $y^2 - 4x = 0 \begin{cases} y = \sqrt{4x} \\ y = -\sqrt{4x} \end{cases}$

x	0	1	4
y	0	2	4

x	0	1	4
$y=-\sqrt{4x}$	0	-2	-4



$x^2 - 4y = 0 \Rightarrow x^2 = 4y \Rightarrow y = \frac{x^2}{4}$

x	-2	0	2
y	1	0	1

$\boxed{\text{P. corte de las dos curvas}}$

$\left.\begin{array}{l} y^2 = 4x \\ x^2 - 4y = 0 \end{array}\right\} \; y = \frac{x^2}{4}$ $\quad \left(\frac{x^2}{4}\right)^2 = 4x \Rightarrow \frac{x^4}{16} = 4x \Rightarrow x^4 = 64x \Rightarrow$

$x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0 \begin{cases} x = 0 \\ x^3 = 64 \Rightarrow x = 4 \end{cases}$

$\int_0^4 \frac{x^2}{4} - \sqrt{4x}\,dx = \frac{1}{4}\int_0^4 x^2\,dx - \sqrt{4}\int_0^4 x^{1/2}\,dx = \left[\frac{1}{4}\,\frac{x^3}{3} - 2\,\frac{x^{3/2}}{3/2}\right]_0^4$

$= \frac{4^3}{4\cdot 3} - \frac{4\cdot\sqrt{4^3}}{3} = \frac{16}{3} - \frac{32}{3} = -\frac{16}{3}$

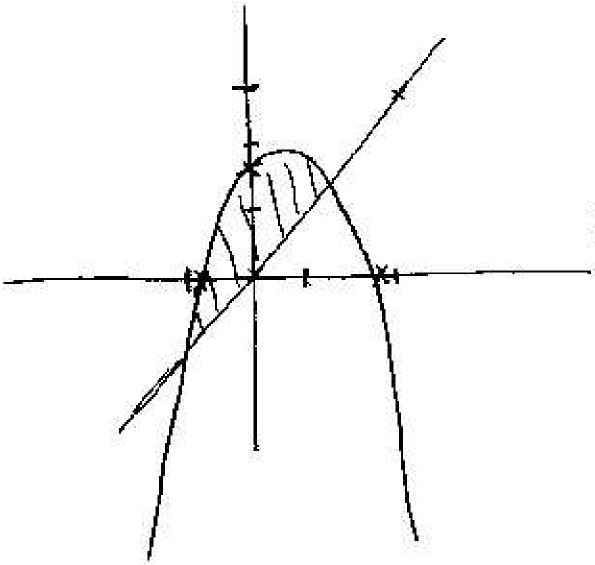
$A = |I| = \left|-\frac{16}{3}\right| = \frac{16}{3}$ u.s.

(14)

Parábola

P. corte con los ejes $-x^2+x+\frac{7}{4}=0 \Rightarrow -4x^2+4x+7=0$

$$x=\frac{-4\pm\sqrt{16+4\cdot4\cdot7}}{-8}=\frac{-4\pm\sqrt{128}}{-8}=\frac{4\pm\sqrt{128}}{8}=\frac{4\pm8\sqrt{2}}{8}\left\langle\begin{array}{l}\frac{1}{2}+\sqrt{2}\simeq 1'9\\ \frac{1}{2}-\sqrt{2}\simeq -0'9\end{array}\right.$$



Recta $y=\frac{3x}{2}-\frac{1}{4}$

x	0	2
y	0	$\frac{11}{4}$

Puntos de corte de parábola y recta

$$\left.\begin{array}{l} y=-x^2+x+\frac{7}{4}\\ y=\frac{3}{2}x-\frac{1}{4}\end{array}\right\} \begin{array}{l} -x^2+x+\frac{7}{4}=\frac{3}{2}x-\frac{1}{4}\\ -x^2+x+\frac{7}{4}-\frac{3}{2}x+\frac{1}{4}=0\\ -x^2-\frac{x}{2}+\frac{8}{4}=0\\ -x^2-\frac{x}{2}+2=0\end{array}$$

$-2x^2-x+4=0$

$$x=\frac{1\pm\sqrt{1+32}}{-4}=\frac{-1\pm\sqrt{33}}{4}$$

$$I=\int_{\frac{-1-\sqrt{33}}{4}}^{\frac{-1+\sqrt{33}}{4}} -x^2+x-\frac{7}{4}-\frac{3x}{2}+\frac{1}{4}\,dx=\int_{\frac{-1-\sqrt{33}}{4}}^{\frac{-1+\sqrt{33}}{4}} -x^2-\frac{x}{2}+2\,dx$$

$$=\left[-\frac{x^3}{3}-\frac{1}{2}\frac{x^2}{2}+2x\right]_{\frac{-1-\sqrt{33}}{4}}^{\frac{-1+\sqrt{33}}{4}}=-\frac{\left(\frac{-1+\sqrt{33}}{4}\right)^3}{3}-\frac{\left(\frac{-1+\sqrt{33}}{4}\right)^2}{4}+2\left(\frac{-1+\sqrt{33}}{4}\right)$$

$$-\left(-\frac{\left(\frac{-1-\sqrt{33}}{4}\right)^3}{3}-\frac{\left(\frac{-1-\sqrt{33}}{4}\right)^2}{4}+2\left(\frac{-1-\sqrt{33}}{4}\right)\right)\simeq$$

$$-\frac{(+1'18)^3}{3}-\frac{(+1'18)^2}{4}+2\cdot1'18-\left(-\frac{(-1'68)^3}{3}-\frac{(1,68)^2}{4}+2(-1'68)\right)=$$

$\simeq 3'94$ u.s.

(13) $f(x) = 3x^2 - 6x.$ $\qquad f'(x) = 6x - 6.$

$$F(x) = \int 3x^2 - 6x \, dx = \frac{3x^3}{3} - \frac{6x^2}{2} + K = x^3 - 3x^2 + K.$$

a) $F(2) = f'(3) \Rightarrow 2^3 - 3 \cdot 2^2 + K = 6 \cdot 3 - 6 \Rightarrow 8 - 12 + K = 18 - 6 \Rightarrow$

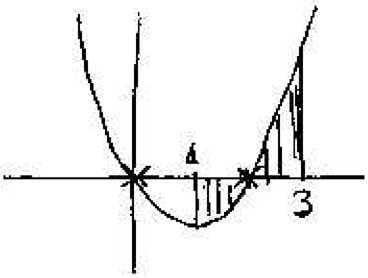
$K = 18 - 6 - 8 + 12 = 16$ $\qquad \boxed{F(x) = x^3 - 3x^2 + 16}$

b) $f(x) = 3x^2 - 6x.$

P. corte con el eje x

$0 = 3x^2 - 6x \Rightarrow x(3x - 6) = 0 \begin{cases} x = 0 \\ 3x - 6 = 0 \Rightarrow x = \frac{6}{3} = 2 \end{cases}$

$$I_1 = \int_1^2 3x^2 - 6x = \left[\frac{3x^3}{3} - \frac{6x^2}{2}\right]_1^2 =$$



$$= \left[x^3 - 3x^2\right]_1^2 = (8 - 12) - (1 - 3) =$$
$$= -4 + 2 = -2$$

$$I_2 = \int_2^3 3x^2 - 6x = \left[x^3 - 3x^2\right]_2^3 = 27 - 27 - (8 - 12) = +4$$

$A = |I_1| + |I_2| = 2 + 4 = 6$ u.s.

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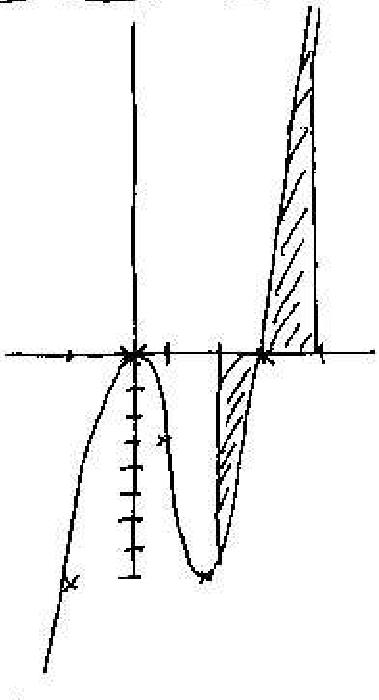
(12) P. corte con los ejes de $y = 2x^3 - 6x^2$

$2x^3 - 6x^2 = 0 \Rightarrow x^2(2x - 6) = 0 \begin{cases} \boxed{x = 0} \\ 2x - 6 = 0 \quad \boxed{x = 3} \end{cases}$

x	$-\infty$	-1	0	1	2	3	4	$+\infty$
y	$-\infty$	-8	0	-4	-8	0	32	$+\infty$



$$I_1 = \int_2^3 2x^3 - 6x^2 = \left[\frac{2x^4}{4} - \frac{6x^3}{3}\right]_2^3 = \left[\frac{x^4}{2} - 2x^3\right]_2^3$$

$$= \left(\frac{81}{2} - 54\right) - (8 - 16) = \frac{81}{2} - 54 + 8 = -\frac{11}{2}$$

$$I_2 = \int_3^4 2x^3 - 6x^2 = \left[\frac{x^4}{2} - 2x^3\right]_3^4 =$$

$$\left(\frac{4^4}{2} - 2 \cdot 4^3\right) - \left(\frac{81}{2} - 54\right) = \frac{219}{2}$$

$A = |I_1| + |I_2| = \frac{11}{2} + \frac{219}{2} = \frac{230}{2} = \boxed{115}$ u.s.

(11) $\int_0^1 \left[x - e^{-2x}\right] dx = \int_0^1 x\,dx - \left(-\frac{1}{2}\right)\int_0^1 \boxed{-2}\, e^{-2x}\,dx$

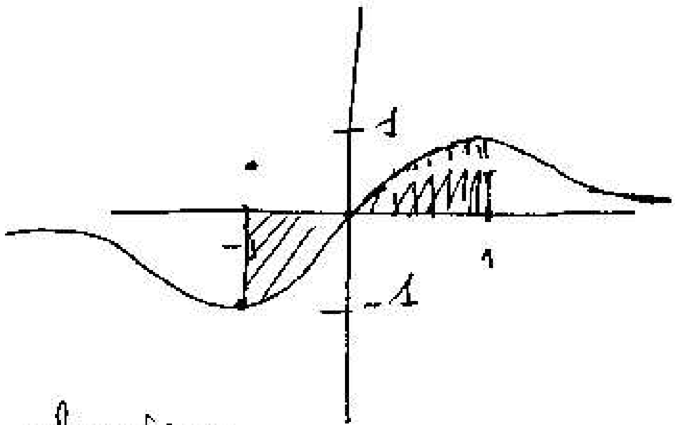
$= \left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\left[e^{-2x}\right]_0^1 = \frac{1}{2} - 0 + \frac{1}{2}e^{-2} - \frac{1}{2}e^0 = \cancel{\frac{1}{2}} + \frac{1}{2e^2} - \cancel{\frac{1}{2}} = \boxed{\frac{1}{2e^2}}$

(12)

P. corte con el eje X $(0,0)$

$\lim_{x\to\infty} \frac{x}{x^2+1} = 0$

x	$-\infty$	-1	0	1	$+\infty$
$y = \frac{x}{x^2+1}$	0	$-1/2$	0	$1/2$	$+\infty$.



La curva es simétrica respecto al origen.

$I = \int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_0^1 \frac{\boxed{2}x}{x^2+1}\,dx = \left[\frac{1}{2}\ln(x^2+1)\right]_0^1 =$

$= \frac{1}{2}\ln 2 - \frac{1}{2}\ln(1) = \frac{\ln 2}{2}$ u.s // $A = 2I = 2\cdot\frac{\ln 2}{2} = \boxed{\ln 2}$ u.s

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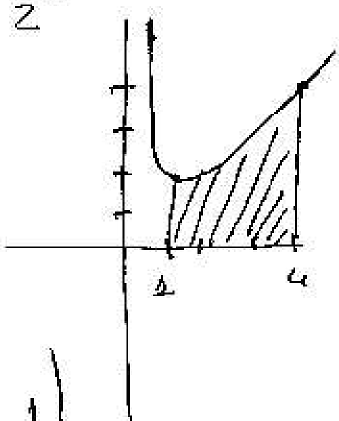
(9) $F(x) = \int x + \frac{1}{x^2}\,dx = \int x + x^{-2}\,dx = \frac{x^2}{2} + \frac{x^{-1}}{-1} + k = \frac{x^2}{2} - \frac{1}{x} + k.$

$F(2) = 5 \Rightarrow \frac{2^2}{2} - \frac{1}{2} + k = 5 \Rightarrow k = 5 - 2 + \frac{1}{2} = \frac{7}{2}$

$\boxed{F(x) = \frac{x^2}{2} - \frac{1}{x} + \frac{7}{2}}$



$\lim_{x\to 0^+} x + \frac{1}{x^2} = 0 + \frac{1}{0^+} = +\infty$

x	0^+	1	4
y	$+\infty$	2	$4+\frac{1}{16}$

$I = \int_1^4 x + \frac{1}{x^2}\,dx = \left[\frac{x^2}{2} + \frac{x^{-1}}{-1}\right]_1^4 = \left[\frac{x^2}{2} - \frac{1}{x}\right]_1^4 = \frac{16}{2} - \frac{1}{4} - \left(\frac{1}{2} - 1\right)$

$8 - \frac{1}{4} - \frac{1}{2} + 1 = \frac{33}{4}$ u.S.

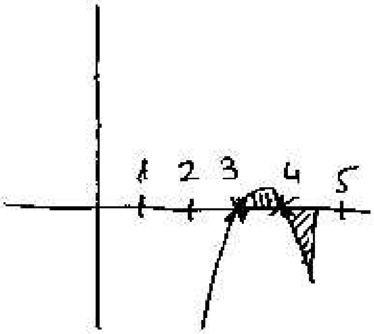
8) $F(x) = \int -x^2+7x-12 \, dx = -\frac{x^3}{3}+\frac{7x^2}{2}-12x+k.$

$F(6)=-2 \Rightarrow -\frac{6^3}{3}+\frac{7\cdot 6^2}{2}-12\cdot 6+k=-2 \Rightarrow -18+k=-2 \Rightarrow k=16.$

$F(x)=\boxed{-\frac{x^3}{3}+\frac{7x^2}{2}-12x+16}$

b) P. corte de la curva con el eje X

$\left.\begin{array}{l} y=-x^2+7x-12 \\ y=0 \end{array}\right\} \; 0=-x^2+7x-12 \qquad x=\frac{-7\pm\sqrt{49-48}}{-2}= \begin{cases} \frac{-7+1}{-2}=3 \\ \frac{-7-1}{2}=4. \end{cases}$



$I_1=\int_3^4 -x^2+7x-12 = \left[-\frac{x^3}{3}+\frac{7x^2}{2}-12x\right]_3^4$

$= -\frac{4^3}{3}+\frac{7\cdot 4^2}{2}-12\cdot 4-\left(-\frac{3^3}{3}+\frac{7\cdot 3^2}{2}-12\cdot 3\right)=\frac{1}{6}$

$I_2=\int_4^{4'5} -x^2+7x-12=\left[-\frac{x^3}{3}+\frac{7x^2}{2}-12x\right]_4^{4'5}=$

$=-\frac{4{,}5^3}{3}+\frac{7\cdot 4{,}5^2}{2}-12\cdot 4'5-\left(-\frac{4^3}{3}+\frac{7\cdot 4^2}{2}-12\cdot 4\right)=\frac{-1}{6}$

$A=|I_1|+|I_2|=\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$ u.s.

7) $\int_1^3 \frac{1}{\sqrt{x}}-\frac{1}{x}\,dx=\int_1^3 x^{-1/2}-\frac{1}{x}\,dx=\left[\frac{x^{-1/2+1}}{-1/2+1}-\ln|x|\right]_1^3=$

$=\left[\frac{x^{1/2}}{1/2}-\ln|x|\right]_1^3=\left[2\sqrt{x}-\ln x\right]_1^3=2\sqrt{3}-\ln 3-(2\sqrt{1}-\ln 1)=$

$=\boxed{2\sqrt{3}-\ln 3-2}$

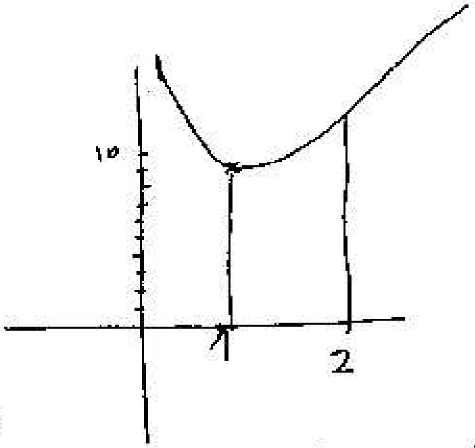
(6) $f(x) = 3x^2 + \frac{2}{x^3} + 5 \quad (x>0)$

$$F(x) = \int 3x^2 + 2x^{-3} + 5\,dx = \frac{3x^3}{3} + 2\frac{x^{-2}}{-2} + 5x + k =$$

$$= x^3 - \frac{1}{x^2} + 5x + k.$$

$$F(x) = x^3 - \frac{1}{x^2} + 5x + 1$$

$$F(1) = 1 - 1 + 5 + k = 6 \Rightarrow k = 1$$



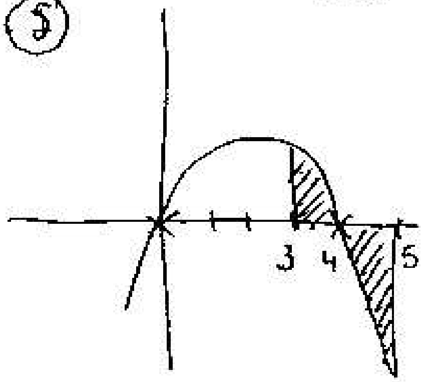
b) $\lim\limits_{x\to 0^+} 3x^2 + \frac{2}{x^3} + 5 = \frac{2}{0^+} + 5 = +\infty.$

x	0^+	1	2	$+\infty$
y	$+\infty$	10	$19+\frac{1}{4}$	$+\infty$

$$I = \int_1^2 3x^2 + \frac{2}{x^3} + 5 = \left[x^3 - \frac{1}{x^2} + 5x\right]_1^2 = 8 - \frac{1}{4} + 10 - (1 - 1 + 5) =$$

$$= 13 - \frac{1}{4} = \boxed{\frac{51}{4}} \text{ u.s.}$$

(5)



$$I_1 = \int_3^4 4x - x^2 = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_3^4 = \left[2x^2 - \frac{x^3}{3}\right]_3^4 =$$

$$= 32 - \frac{64}{3} - \left(18 - \frac{27}{3}\right) = 32 - \frac{64}{3} - 9 = 23 - \frac{64}{3} = \frac{69-64}{3} = \frac{5}{3}$$

$$I_2 = \int_4^5 4x - x^2 = \left[2x^2 - \frac{x^3}{3}\right]_4^5 = \left(50 - \frac{125}{3}\right) - \left(33 - \frac{64}{3}\right) =$$

$$= 50 - \frac{125}{3} - 33 + \frac{64}{3} = 17 - \frac{61}{3} = \frac{51-61}{3} = -\frac{10}{3}$$

$$A = |I_1| + |I_2| = \frac{5}{3} + \frac{10}{3} = \frac{15}{3} = 5 \text{ u.s.}$$

(4) $F(x) = \int x^2 + bx = \frac{x^3}{3} + b\frac{x^2}{2} + k.$

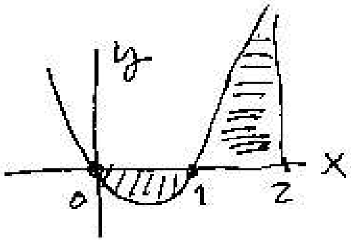
$F(0) = 2 \Rightarrow 0 + 0 + k = 2$

$F(3) = 20 \Rightarrow \frac{27}{3} + b\frac{9}{2} + k = 20$

$$\begin{cases} 9 + \frac{9}{2}b + 2 = 20 \\ \frac{9}{2}b = 9 \Rightarrow 9b = 18 \Rightarrow b = 2 \end{cases}$$

$$F(x) = \frac{x^3}{3} + 2\frac{x^2}{2} + 2 = \frac{x^3}{3} + x^2 + 2$$

(4b) $b = -1 \rightarrow y = x^2 - x$



$$I_1 = \int_0^1 x^2 - x = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

$$I_2 = \int_1^2 x^2 - x = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2 = \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{8}{3} - 2 + \frac{1}{6} = \frac{16 - 12 + 1}{6} = \frac{5}{6}$$

$$A = |I_1| + |I_2| = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1$$

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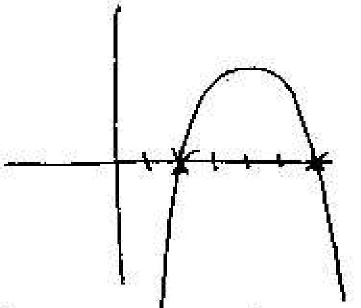
(3) P. corte de la curva con el eje x



$$\left.\begin{array}{l} y = 8x - x^2 - 12 \\ y = 0 \end{array}\right\} \begin{array}{l} 0 = 8x - x^2 - 12 = -x^2 + 8x - 12 \\ x = \dfrac{-8 \pm \sqrt{64 - 48}}{-2} = \dfrac{-8 \pm 4}{-2} \left< \begin{array}{l} 2 \\ 6 \end{array}\right. \end{array}$$

$$I = \int_{-1}^2 8x - x^2 - 12 = \left[\frac{8x^2}{2} - \frac{x^3}{3} - 12x\right]_{-1}^2 = \left(16 - \frac{8}{3} - 24\right) - \left(4 + \frac{1}{3} + 12\right) =$$

$$= \cancel{16} - \frac{8}{3} - 24 - \cancel{4} - \frac{1}{3} - \cancel{12} = -\frac{9}{3} - 24 = -3 - 24 = -27$$

$$A = |I_1| = 27$$

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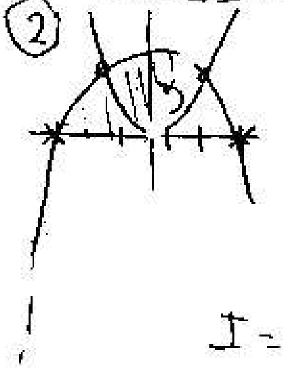
(2)



$$D = \int_{-3}^3 9 - x^2 = \left[9x - \frac{x^3}{3}\right]_{-3}^3 = 27 - \frac{27}{3} - \left(-27 + \frac{27}{3}\right) = \frac{108}{3}$$

P. corte de $\left.\begin{array}{l} y = 9 - x^2 \\ y = 8x^2 \end{array}\right\} 9 - x^2 = 8x^2 \Rightarrow 9 = 9x^2 \Rightarrow x = \pm 1$

$$I = \int_{-1}^1 9 - x^2 - 8x^2 = \int_{-1}^1 9 - 9x^2 = \left[9x - \frac{9x^3}{3}\right]_{-1}^1 = \left[9x - 3x^3\right]_{-1}^1 = 6 + 6 = 12 \text{ u.s.}$$

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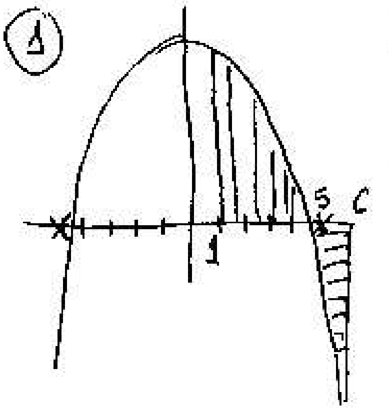
(1)



$$I_1 = \int_1^5 25 - x^2 = \left[25x - \frac{x^3}{3}\right]_1^5 = 125 - \frac{125}{3} - (25 - 1) = \frac{178}{3}$$

$$I_2 = \int_5^6 25 - x^2 = \left[25x - \frac{x^3}{3}\right]_5^6 = 150 - \frac{216}{3} - \left(125 - \frac{125}{3}\right) = -\frac{16}{3}$$

$$A = \left|\frac{178}{3}\right| + \left|-\frac{16}{3}\right| = \frac{178}{3} + \frac{16}{3} = \frac{194}{3} \text{ u.s.}$$