Todas las técnicas de integración consisten en transformar el integrando hasta obtener una función que reconozcamos como inmediata. Por ello, el conocimiento y memorización de los siguientes tipos es imprescindible para iniciarse en la integración.

INTEGRALES DE FUNCIONES ELEMENTALES:

TIPOS	FORMAS	
	SIMPLES	COMPUESTAS
Potencial α≠–1	$\int x^{\alpha}.dx = \frac{x^{\alpha+1}}{\alpha+1} + K$	$\int f^{\alpha}.f'.dx = \frac{f^{\alpha+1}}{\alpha+1} + K$
Logarítmico	$\int \frac{1}{x}\cdot dx = L\lvert x\rvert + K$	$\int \frac{f'}{f}\cdot dx = L\lvert f\rvert + K$
Exponencial	$\int e^{x}dx = e^{x} + K$	$\int f'.e^{f}dx = e^{f} + K$
	$\int a^{x}dx = \frac{1}{La}\cdot a^{x} + K$	$\int f'.a^{f}dx = \frac{1}{La}\cdot a^{f} + K$
Seno	$\int \cos x.dx = \operatorname{sen} x + K$	$\int f'.\cos f.dx = \operatorname{sen} f + K$
Coseno	$\int \operatorname{sen} x.dx = -\cos x + K$	$\int f'.\operatorname{sen} f.dx = -\cos f + K$
Tangente	$\int \sec^{2} x.dx = \operatorname{tg} x + K$	$\int f'\sec^{2} f.dx = \operatorname{tg} f + K$
	$\int (1+\operatorname{tg}^{2} x)dx = \operatorname{tg} x + K$	$\int (1+\operatorname{tg}^{2} f)f'dx = \operatorname{tg} f + K$
	$\int \frac{1}{\cos^{2} x}\cdot dx = \operatorname{tg} x + K$	$\int \frac{f'}{\cos^{2} f}\cdot dx = \operatorname{tg} f + K$
Cotangente	$\int \operatorname{cosec}^{2}x.dx = -\operatorname{ctg} x + K$	$\int f'.\operatorname{cosec}^{2} f.dx = -\operatorname{ctg} f + K$
	$\int (1+\operatorname{ctg}^{2}x)dx = -\operatorname{ctg} x + K$	$\int (1+\operatorname{tg}^{2} f)f'dx = \operatorname{tg} f + K$
	$\int \frac{1}{\operatorname{sen}^{2} x}\cdot dx = -\operatorname{ctg} x + K$	$\int \frac{f'}{\operatorname{sen}^{2} f}\cdot dx = -\operatorname{ctg} f + K$
Arco seno (= –arco coseno)	$\int \frac{1}{\sqrt{1-x^{2}}}\cdot dx = \operatorname{arcsen} x + K =$ $= -\arccos x + K$	$\int \frac{f'}{\sqrt{1-f^{2}}}\cdot dx = \operatorname{arcsen} f + K =$ $= -\arccos f + K$
	$\int \frac{1}{\sqrt{a^{2}-x^{2}}}\cdot dx = \operatorname{arcsen}\frac{x}{a} + K$ $= -\arccos\frac{x}{a} + K$	$\int \frac{f'}{\sqrt{a^{2}-f^{2}}}\cdot dx = \operatorname{arcsen}\frac{f}{a} + K$ $= -\arccos\frac{f}{a} + K$
Arco tangente **= –Arco cotangente.**	$\int \frac{1}{1+x^{2}}\cdot dx = \operatorname{arctg} x + K$ $= -\operatorname{arcctg} x + K$	$\int \frac{f'}{1+f^{2}}\cdot dx = \operatorname{arctg} f + K$ $= -\operatorname{arcctg} f + K$
	$\int \frac{1}{a^{2}+x^{2}}\cdot dx = \frac{1}{a}\cdot \operatorname{arctg}\frac{x}{a} + K$ $= -\frac{1}{a}\cdot \operatorname{arcctg}\frac{x}{a} + K$	$\int \frac{f'}{a^{2}+f^{2}}\cdot dx = \frac{1}{a}\cdot \operatorname{arctg}\frac{f}{a} + K$ $= -\frac{1}{a}\cdot \operatorname{arcctg}\frac{f}{a} + K$

EJEMPLOS:

Tipo potencial: forma simple

- $\int x^5 dx = \dfrac{x^{5+1}}{5+1} + K = \dfrac{x^6}{6} + K$

- $\int x^{\frac{2}{3}}.dx = \dfrac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + K = \dfrac{x^{\frac{5}{3}}}{\frac{5}{3}} + K = \dfrac{3x^{\frac{5}{3}}}{5} + K$

- $\int \sqrt{x^3}dx = \int x^{\frac{3}{2}}.dx = \dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + K = \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + K = \dfrac{2\sqrt{x^5}}{5} + K = \dfrac{2x^2\sqrt{x}}{5} + K$

- $\int \dfrac{1}{\sqrt[4]{x}} \cdot dx = \int \dfrac{1}{x^{1/4}} \cdot dx = \int x^{-\frac{1}{4}}.dx = \dfrac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + K = \dfrac{x^{\frac{3}{4}}}{\frac{3}{4}} + K = \dfrac{4\sqrt[4]{x^3}}{3} + K = \dfrac{4}{3}\sqrt[4]{x^3} + K$

- $\int \dfrac{x^2}{\sqrt{x}} \cdot dx = \int x^2.x^{-\frac{1}{2}}.dx = \int x^{\frac{3}{2}}.dx = \dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + K = \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + K = \dfrac{2\sqrt{x^5}}{5} + K = \dfrac{2x^2\sqrt{x}}{5} + K$

- Combinando la integral inmediata de tipo potencial con las propiedades lineales de la integral indefinida, podemos integrar funciones de tipo polinómico:

$$\int (2x^3 + 5x^2 - 7x + 3).dx = \int 2x^3.dx + \int 5x^2.dx - \int 7x.dx + \int 3.dx =$$

$$= 2\int x^3.dx + 5\int x^2.dx - 7\int x.dx + 3\int dx =$$

$$= 2 \cdot \frac{x^{3+1}}{3+1} + 5 \cdot \frac{x^{2+1}}{2+1} - 7 \cdot \frac{x^{1+1}}{1+1} + 3x + K =$$

$$= 2 \cdot \frac{x^4}{4} + 5 \cdot \frac{x^3}{3} - 7 \cdot \frac{x^2}{2} + 3x + K = \frac{x^4}{2} + \frac{5x^3}{3} - \frac{7x^2}{2} + 3x + K$$

Vemos que el proceso de integración lo hemos aplicado a las funciones potenciales dejando los coeficientes al margen del proceso. Sin embargo, no hace falta dar todos los pasos como en el ejemplo anterior, sino que se puede y se debe integrar directamente como en el siguiente ejemplo:

$$\int (2x^5 - \frac{2}{3}x^2 + 3x - 7).dx = 2\cdot\frac{x^6}{6} - \frac{2}{3}\cdot\frac{x^3}{3} + 3\cdot\frac{x^2}{2} - 7x + K =$$
$$= \frac{1}{3}\cdot x^6 - \frac{2}{9}\cdot x^3 + \frac{3}{2}\cdot x^2 - 7x + K$$

- $$\int (x+\sqrt{x}).dx = \int (x + x^{\frac{1}{2}}).dx = \int x.dx + \int x^{\frac{1}{2}}.dx = \frac{x^{1+1}}{1+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + K =$$
$$= \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + K = \frac{x^2}{2} + \frac{2x^{\frac{3}{2}}}{3} + K$$

- $$\int \left(x^2 + \frac{1}{\sqrt[3]{x}} \right)^2 dx = \int \left(x^2 + x^{-\frac{1}{3}} \right)^2 dx = \int (x^4 + 2x^{\frac{5}{3}} + x^{-\frac{2}{3}}).dx = \frac{x^5}{5} + 2\cdot\frac{x^{\frac{8}{3}}}{\frac{8}{3}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + K =$$
$$= \frac{1}{5}\cdot x^5 + \frac{3}{4}\cdot x^{\frac{8}{3}} + 3.x^{\frac{1}{3}} + K = \frac{1}{5}\cdot x^5 + \frac{3}{4}\cdot\sqrt[3]{x^8} + 3.\sqrt[3]{x} + K =$$
$$= \frac{1}{5}\cdot x^5 + \frac{3}{4}\cdot x^2.\sqrt[3]{x^2} + 3.\sqrt[3]{x} + K$$

- $$\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right)\cdot dx = \int (x^{-2} + 4x^{-\frac{3}{2}} + 2).dx = \frac{x^{-1}}{-1} + 4\cdot\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 2x + K = -\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + K$$

<u>Tipo potencial:</u> forma compuesta.

- $$\int (x+2)^3.dx = \left\{ \int f^3.f' \right\} = \frac{(x+2)^4}{4} + K$$
- $$\int (2x+1).(x^2+x+1)^{30}.dx = \left\{ \int f'.f^{30} \right\} = \frac{(x^2+x+1)^{31}}{31} + K$$
- $$\int x.(x^2+1).dx = \int \frac{1}{2}\cdot 2x.(x^2+1).dx = \frac{1}{2}\int \underbrace{2x}_{f'}.\underbrace{(x^2+1)}_{f}.dx = \frac{1}{2}\cdot\frac{(x^2+1)^2}{2} + K = \frac{(x^2+1)^2}{4} + K$$
- $$\int \mathrm{sen}^3 x.\cos x.dx = \left\{ \int f^3.f' \right\} = \frac{\mathrm{sen}^4 x}{4} + K$$
- $$\int \mathrm{tg}^2 x.\sec^2 x.dx = \left\{ \int f^2.f' \right\} = \frac{\mathrm{tg}^3 x}{3} + K$$
- $$\int (\mathrm{tg}^3 x + \mathrm{tg}^5 x).dx = \int \underbrace{\mathrm{tg}^3 x}_{f^3}\cdot\underbrace{(1+\mathrm{tg}^2 x)}_{f'}.dx = \frac{\mathrm{tg}^4 x}{4} + K$$
- $$\int x^2.\sqrt{x^3-1}.dx = \int x^2.\underbrace{(x^3-1)}_{f}^{\frac{1}{2}}.dx = \int \frac{1}{3}\cdot\underbrace{3x^2}_{f'}.\underbrace{(x^3-1)}_{f}^{\frac{1}{2}} dx = \frac{1}{3}\int \underbrace{3x^2}_{f'}.\underbrace{(x^3-1)}_{f}^{\frac{1}{2}}.dx =$$

$$= \frac{1}{3} \cdot \frac{(x^3-1)^{\frac{3}{2}}}{\frac{3}{2}} + K = \frac{2}{9}\sqrt{(x^3-1)^3} + K = \frac{2}{9} \cdot (x^3-1)\sqrt{x^3-1} + K$$

Tipo logarítmico:

- $\int \frac{3}{x+2} \cdot dx = 3\int \frac{1}{x+2} \cdot dx = 3 \cdot L|x+2| + K$
- $\int \frac{x}{x^2+1} \cdot dx = \frac{1}{2} \cdot \int \frac{2x}{x^2+1} \cdot dx = \frac{1}{2} \cdot L(x^2+1) + K$
- $\int \operatorname{tg} x \cdot dx = \int \frac{\operatorname{sen} x}{\cos x} \cdot dx = -\int \frac{-\operatorname{sen} x}{\cos x} \cdot dx = -L|\cos x| + K$
- $\int \operatorname{ctg} x \cdot dx = \int \frac{\cos x}{\operatorname{sen} x} \cdot dx = L|\operatorname{sen} x| + K$
- $\int \frac{\operatorname{sen} 2x}{1+\operatorname{sen}^2 x} \cdot dx = \int \frac{2\operatorname{sen} x.\cos x}{1+\operatorname{sen}^2 x} \cdot dx = L(1+\operatorname{sen}^2 x) + K$
- $\int \frac{1}{(1+x^2).\operatorname{arctg} x} \cdot dx = \int \frac{\frac{1}{1+x^2}}{\operatorname{arctg} x} \cdot dx = L|\operatorname{arctg} x| + K$
- $\int \frac{1}{\operatorname{arcsen} x} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = \int \frac{\frac{1}{\sqrt{1-x^2}}}{\operatorname{arcsen} x} \cdot dx = L|\operatorname{arcsen} x| + K$

Tipo exponencial:

- $\int 3^x.dx = \frac{1}{L3} \cdot \int 3^x \cdot L3 \cdot dx = \frac{1}{L3} \cdot 3^x + K$
- $\int e^{x+1} \cdot dx = \int 1 \cdot e^{x+1} \cdot dx = e^{x+1} + K$
- $\int e^{2x+1} \cdot dx = \frac{1}{2}\int 2 \cdot e^{2x+1} \cdot dx = \frac{1}{2} e^{2x+1} + K$
- $\int 2^x \cdot 3^x.dx = \int (2 \cdot 3)^x.dx = \frac{1}{L(2.3)} \cdot \int (2 \cdot 3)^x \cdot L(2 \cdot 3) \cdot dx = \frac{1}{L(2 \cdot 3)} \cdot (2 \cdot 3)^x + K$
- $\int x \cdot e^{x^2+1} \cdot dx = \frac{1}{2}\int 2x \cdot e^{x^2+1} \cdot dx = \frac{1}{2} e^{x^2+1} + K$
- $\int e^{\operatorname{sen} x} \cdot \cos x \cdot dx = e^{\operatorname{sen} x} + K$
- $\int e^{\operatorname{sen}^2 x} \cdot \operatorname{sen} 2x \cdot dx = \int e^{\operatorname{sen}^2 x} \cdot 2\operatorname{sen} x.\cos x \cdot dx = e^{\operatorname{sen}^2 x} + K$
- $\int \frac{e^{\operatorname{arcsen} x}}{\sqrt{1-x^2}} \cdot dx = \int \frac{1}{\sqrt{1-x^2}} \cdot e^{\operatorname{arcsen} x} \cdot dx = e^{\operatorname{arcsen} x} + K$
- $\int \frac{e^{\operatorname{arctg} x}}{1+x^2} \cdot dx = \int \frac{1}{1+x^2} \cdot e^{\operatorname{arctg} x} dx = e^{\operatorname{arctg} x} + K$

Tipo trigonométrico (seno, coseno, tangente,....).

- $\int \cos(2x-1)\cdot dx = \frac{1}{2}\cdot\int 2\cdot\cos(2x-1)\cdot dx = \frac{1}{2}\cdot \text{sen}(2x-1)+K$
- $\int x\cdot\cos x^2\cdot dx = \frac{1}{2}\cdot\int 2\cdot\cos x^2\cdot dx = \frac{1}{2}\cdot\text{sen}\,x^2+K$
- $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot dx = 2\int \frac{1}{2\sqrt{x}}\cdot\cos\sqrt{x}\cdot dx = 2\cdot\text{sen}\sqrt{x}+K$
- $\int \frac{\cos(Lx)}{x}\cdot dx = \int \frac{1}{x}\cdot\cos(Lx)\cdot dx = \text{sen}(Lx)+K$
- $\int \text{sen}\,2x\cdot dx = \frac{1}{2}\cdot\int 2\cdot\text{sen}\,2x\cdot dx = -\frac{1}{2}\cdot\cos 2x+K$
- $\int \text{sen}\,2x\cdot dx = \int 2\cdot\text{sen}\,x\cdot\cos x\cdot dx = 2\cdot\int \underbrace{\text{sen}\,x}_{f}\cdot\underbrace{\cos x}_{f'}\cdot dx = 2\cdot\frac{\text{sen}^2 x}{2}+K = \text{sen}^2 x+K$
- $\int x\cdot\text{sen}(x^2+3)\cdot dx = \frac{1}{2}\cdot\int 2x\cdot\text{sen}(x^2+3)\cdot dx = -\frac{1}{2}\cdot\cos(x^2+3)+K$
- $\int \frac{\text{sen}\sqrt{x}}{\sqrt{x}}\cdot dx = 2\int \frac{1}{2\sqrt{x}}\cdot\text{sen}\sqrt{x}\cdot dx = -2\cdot\cos\sqrt{x}+K$
- $\int e^x\cdot\text{sen}(e^x+3)\cdot dx = -\cos(e^x+3)+K$
- $\int \text{tg}^2 x.dx = \int (1+\text{tg}^2 x-1).dx = \int (1+\text{tg}^2 x).dx - \int 1.dx = \text{tg}\,x - x + K$
- $\int \text{tg}^3 x.dx = \int \text{tg}\,x\cdot\text{tg}^2 x.dx = \int \text{tg}\,x(\sec^2 x-1).dx = \int (\text{tg}\,x\cdot\sec^2 x - \text{tg}\,x).dx =$

$$= \int \text{tg}\,x\cdot\sec^2 x.dx - \int \text{tg}\,x.dx = \frac{\text{tg}^2 x}{2} - \int \frac{\text{sen}\,x}{\cos x}\cdot dx =$$

$$= \frac{\text{tg}^2 x}{2} - (-L|\cos x|)+K = \frac{\text{tg}^2 x}{2} + L|\cos x| + K$$

- $\int \sec^2(3x-1)\cdot dx = \frac{1}{3}\cdot\int 3\sec^2(3x-1)\cdot dx = \frac{1}{3}\cdot\text{tg}(3x-1)+K$
- $\int \frac{1}{\text{sen}^2 7x}\cdot dx = \frac{1}{7}\cdot\int \frac{7}{\text{sen}^2 7x}\cdot dx = -\frac{1}{7}\cdot\text{ctg}\,7x+K$
- $\int \frac{3x}{\cos^2 2x^2}\cdot dx = 3\cdot\frac{1}{4}\cdot\int \frac{4x}{\cos^2 2x^2}\cdot dx = \frac{3}{4}\cdot\text{tg}\,2x^2+K$
- $\int \text{ctg}^2 x.dx = \int (1+\text{ctg}^2 x-1).dx = \int (1+\text{ctg}^2 x).dx - \int 1.dx = -\text{ctg}\,x - x + K$