SOLUCIONES TEMA 5 - 1º BCN

11.- $\sin x = \frac{3}{5}$; $\pi/2 < x < \pi$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} \Rightarrow \cos^2 x = \frac{16}{25} \Rightarrow \cos x = \pm\frac{4}{5} \Rightarrow$ La solución es

$\cos x = -\frac{4}{5}$ para que el ángulo esté entre $\pi/2$ y π; $\tan x = \frac{3/5}{-4/5} \Rightarrow \tan x = -\frac{3}{4}$

a) $\sin 2x = 2\cdot\frac{3}{5}\cdot\left(-\frac{4}{5}\right) = -\frac{24}{25}$

b) $\tan\frac{x}{2} = \sqrt{\frac{1+4/5}{1-4/5}} = \sqrt{9} = 3$ La solución es positiva pues $x/2$ ha de ser agudo

c) $\sin(x+\pi/6) = \frac{3}{5}\cdot\cos\frac{\pi}{6} + \frac{-4}{5}\sin\frac{\pi}{6} = \frac{3}{5}\cdot\frac{\sqrt{3}}{2} - \frac{4}{5}\cdot\frac{1}{2} = \frac{3\sqrt{3}-4}{10}$

d) $\cos(x-\pi/3) = -\frac{4}{5}\cos\frac{\pi}{3} + \frac{3}{5}\sin\frac{\pi}{3} = -\frac{4}{5}\cdot\frac{1}{2} + \frac{3}{5}\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}-4}{10}$

e) $\cos\frac{x}{2} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$ La solución es positiva pues $x/2$ es agudo

f) $\tan(x+\pi/4) = \frac{-3/4 + \tan\pi/4}{1+3/4\cdot\tan\pi/4} = \frac{-3/4+1}{1+3/4\cdot 1} = \frac{1/4}{7/4} = \frac{1}{7}$

18.- a)

$2\cos^x - \sin^2 x + 1 = 0$; haciendo $\sin^2 x = 1 - \cos^2 x \Rightarrow$

$2\cos^2 x - 1 + \cos^2 x + 1 = 0 \Rightarrow 3\cos^2 x = 0 \Rightarrow \cos x = 0 \Rightarrow \begin{matrix} x = 90^\circ \\ x = 270^\circ \end{matrix}$

c)

$2\cos^2 x - \sqrt{3}\cos x = 0 \Rightarrow \cos x(2\cos x - \sqrt{3}) = 0 \Rightarrow \begin{cases} \cos x = 0 \Rightarrow \begin{matrix} x = 90^\circ \\ x = 270^\circ \end{matrix} \\ 2\cos x - \sqrt{3} = 0 \Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow \begin{matrix} x = 30^\circ \\ x = 330^\circ \end{matrix} \end{cases}$

21.- a) $4\sin^2 x\cos^2 x + 2\cos^2 x - 2 = 0 \Rightarrow 4(1-\cos^2 x)\cos^2 x + 2\cos^2 x - 2 = 0 \Rightarrow$

$4\cos^2 x - 4\cos^4 x + 2\cos^2 x - 2 = 0 \Rightarrow 4\cos^4 x - 6\cos^2 x + 2 = 0 \Rightarrow$ Haciendo $\cos^2 x = t \Rightarrow$

$$2t^2 - 3t + 1 = 0 \Rightarrow \begin{cases} t = 1 \Rightarrow \cos^2 x = 1 \Rightarrow \cos x = \pm 1 \Rightarrow \begin{matrix} x = 0^\circ \\ x = 180^\circ \end{matrix} \\ t = 1/2 \Rightarrow \cos^2 x = 1/2 \Rightarrow \cos x = \pm\sqrt{2}/2 \Rightarrow \begin{matrix} x = 45^\circ;\ x = 315^\circ \\ x = 135^\circ;\ x = 225^\circ \end{matrix} \end{cases}$$

b) $4\sin^2 x + \sin x \cos x - 3\cos^2 x = 0 \Rightarrow \dfrac{4\sin^2 x}{\cos^2 x} + \dfrac{\sin x \cos x}{\cos^2 x} - \dfrac{3\cos^2 x}{\cos^2 x} = 0 \Rightarrow$

$$4\tan^2 x + \tan x - 3 = 0 \Rightarrow \begin{cases} \tan x = -1 \Rightarrow \begin{matrix} x = 135^\circ \\ x = 315^\circ \end{matrix} \\ \tan x = 3/4 \Rightarrow \begin{matrix} x \approx 37^\circ \\ x \approx 217^\circ \end{matrix} \end{cases}$$

c) $\cos^2 \dfrac{x}{2} + \cos x - \dfrac{1}{2} = 0 \Rightarrow \dfrac{1 + \cos x}{2} + \cos x - \dfrac{1}{2} = 0 \Rightarrow 1 + 3\cos x - 1 = 0 \Rightarrow$

$\cos x = 0 \Rightarrow \begin{matrix} x = 90^\circ \\ x = 270^\circ \end{matrix}$

d) $\tan^2 \dfrac{x}{2} + 1 = \cos x \Rightarrow \dfrac{1 - \cos x}{1 + \cos x} + 1 = \cos x \Rightarrow 1 - \cos x + 1 + \cos x = \cos x + \cos^2 x \Rightarrow$

$\cos^2 x + \cos x - 2 = 0 \Rightarrow \cos x = 1 \Rightarrow x = 0^\circ$

e) $2\sin^2 \dfrac{x}{2} + \cos 2x = 0 \Rightarrow 2 \cdot \dfrac{1 - \cos x}{2} + \cos^2 x - \sin^2 x = 0 \Rightarrow$

$1 - \cos x + \cos^2 x - 1 + \cos^2 x = 0 \Rightarrow 2\cos^2 x - \cos x = 0 \Rightarrow \cos x(2\cos x - 1) = 0 \Rightarrow$

$\cos x = 0 \Rightarrow x = 90^\circ; \quad x = 270^\circ$

$2\cos x - 1 = 0 \Rightarrow \cos x = 1/2 \Rightarrow x = 60^\circ; \quad x = 300^\circ$

22.-

$$\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} \Rightarrow$$

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$$

25.- $\cos\alpha \cdot \cos(\alpha - \beta) + \sin\alpha \cdot \sin(\alpha - \beta) = \cos\beta \Rightarrow$

$\cos\alpha\cdot(\cos\alpha\cos\beta+\sin\alpha\sin\beta)+\sin\alpha\cdot(\sin\alpha\cos\beta-\cos\alpha\sin\beta)=$

$\cos^2\alpha\cos\beta+\cos\alpha\sin\alpha\sin\beta+\sin^2\alpha\cos\beta-\cos\alpha\sin\alpha\sin\beta=\cos\beta\cdot(\cos^2\alpha+\sin^2\alpha)=\cos\beta\cdot 1=\cos\beta$

27.- 1 radián es el ángulo cuyo arco es igual al radio.

$$\left.\begin{array}{ccc} 12cm & - & 2{,}5rad \\ xcm & - & 1rad \end{array}\right\} \Rightarrow x=\frac{12}{2{,}5}=4{,}8cm \text{ de radio}$$

30.- $\dfrac{\sin 2\alpha}{1-\cos^2\alpha}=\dfrac{2\sin\alpha\cos\alpha}{\sin^2\alpha}=\dfrac{2\cos\alpha}{\sin\alpha}=\dfrac{2}{\tan\alpha};\qquad \dfrac{2}{\tan\pi/4}=\dfrac{2}{1}=2$

33.- a) $\cos 2x+3\sin x=2\Rightarrow\cos^2 x-\sin^2 x+3\sin x=2\Rightarrow$

$$1-\sin^2 x-\sin^2 x+3\sin x=2\Rightarrow 2\sin^2 x-3\sin x+1=0\Rightarrow \begin{array}{l} \sin x=1\Rightarrow x=90^\circ \\ \sin x=1/2\Rightarrow \begin{array}{l} x=30^\circ \\ x=150^\circ \end{array} \end{array}$$

b) $\tan 2x\cdot\tan x=1\Rightarrow\dfrac{2\tan x}{1-\tan^2 x}\cdot\tan x=1\Rightarrow 2\tan^2 x=1-\tan^2 x\Rightarrow 3\tan^2 x=1\Rightarrow$

$$\tan x=\pm\frac{\sqrt{3}}{3}\Rightarrow x=30^\circ;\quad x=210^\circ;\quad x=150^\circ;\quad x=330^\circ$$

c) $\cos x\cos 2x+2\cos^2 x=0\Rightarrow\cos x(\cos^2 x-\sin^2 x)+2\cos^2 x=0\Rightarrow$

$$\cos x(\cos^2 x-1+\cos^2 x+2\cos x)=0\Rightarrow \begin{array}{l} \cos x=0\Rightarrow x=0^\circ;\quad x=270^\circ \\ 2\cos^2 x+2\cos x-1=0 \end{array}$$

$$2\cos^2 x+2\cos x-1=0\Rightarrow \begin{array}{l} \cos x=0{,}37\Rightarrow x=68{,}5^\circ;\quad x=231{,}5^\circ \\ \cos x=-1{,}36\Rightarrow \text{No hay solución para } x \end{array}$$

d) $2\sin x=\tan 2x\Rightarrow 2\sin x=\dfrac{2\tan x}{1-\tan^2 x}\Rightarrow 2\sin x-2\sin x\dfrac{\sin^2 x}{\cos^2 x}=2\dfrac{\sin x}{\cos x}\Rightarrow$

$$2\sin x\cos^2 x-2\sin^3 x=2\sin x\cos x\Rightarrow 2\sin x(\cos x-\cos^2 x+\sin^2 x)=0\Rightarrow$$

$2\sin x=0\Rightarrow x=0^\circ;\quad x=180^\circ$

$$\cos x-\cos^2 x+1-\cos^2 x=0\Rightarrow 2\cos^2 x-\cos x-1=0\Rightarrow \begin{array}{l} \cos x=1\Rightarrow x=0^\circ \\ \cos x=-1/2\Rightarrow x=120^\circ;\quad x=240^\circ \end{array}$$

e) $\sqrt{3}\sin\dfrac{x}{2}+\cos x-1=0\Rightarrow\sqrt{3}\sin\dfrac{x}{2}+\cos^2\dfrac{x}{2}-\sin^2\dfrac{x}{2}-1=0\Rightarrow$

$$\sqrt{3}\sin\frac{x}{2}+1-\sin^2\frac{x}{2}-\sin^2\frac{x}{2}-1=0\Rightarrow 2\sin^2\frac{x}{2}-\sqrt{3}\sin\frac{x}{2}=0\Rightarrow \sin\frac{x}{2}\left(2\sin\frac{x}{2}-\sqrt{3}\right)=0\Rightarrow$$

$$\begin{cases}\sin\frac{x}{2}=0\Rightarrow\frac{x}{2}=0\Rightarrow x=0^\circ\\ 2\sin\frac{x}{2}=\sqrt{3}\Rightarrow\sin\frac{x}{2}=\frac{\sqrt{3}}{2}\Rightarrow \begin{matrix}x/2=60\Rightarrow x=120^\circ\\ x/2=120\Rightarrow x=240^\circ\end{matrix}\end{cases}$$

f) $\sin 2x\cos x=6\sin^3 x\Rightarrow 2\sin x\cos x\cos x=6\sin^3 x\Rightarrow 2\sin x(3\sin^2 x-\cos^2 x)=0\Rightarrow$

$2\sin x=0\Rightarrow\sin x=0\Rightarrow x=0^\circ;\qquad 3\sin^2 x-\cos^2 x=0\Rightarrow 3\sin^2 x-1+\sin^2 x=0\Rightarrow$

$$4\sin^2 x=1\Rightarrow\sin^2 x=1/4\Rightarrow\begin{matrix}\sin x=1/2\Rightarrow x=30^\circ;\quad x=150^\circ\\ \sin x=-1/2\Rightarrow x=330^\circ;\quad x=210^\circ\end{matrix}$$

g) $\tan\left(\frac{\pi}{4}-x\right)+\tan x=1\Rightarrow\frac{\tan(\pi/4)-\tan x}{1+\tan(\pi/4)\tan x}=1\Rightarrow$

$1-\tan x+\tan x+\tan^2 x=1+\tan x\Rightarrow\tan^2 x-\tan x=0\Rightarrow\tan x(\tan x-1)=0\Rightarrow$

$$\begin{matrix}\tan x=x\Rightarrow x=0^\circ;\quad x=180^\circ\\ \tan x-1=0\Rightarrow\tan x=1\Rightarrow x=45^\circ\quad x=225^\circ\end{matrix}$$

34.- a) $\sin 3x-\sin x=\cos 2x\Rightarrow\ 2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}=\cos 2x\Rightarrow$

$$2\cos 2x\sin x=\cos 2x\Rightarrow\cos 2x(2\sin x-1)=0\Rightarrow\begin{matrix}\cos 2x=0\Rightarrow\begin{matrix}2x=90\Rightarrow x=45\\ 2x=270\Rightarrow x=135^\circ\end{matrix}\\ 2\sin x=1\Rightarrow\sin x=1/2\Rightarrow\begin{matrix}x=30^\circ\\ x=150^\circ\end{matrix}\end{matrix}$$

d) $\sin 3x-\cos 3x=\sin x-\cos x\Rightarrow\sin 3x-\sin x=\cos 3x-\cos x\Rightarrow$

$$2\cos 2x\sin x=-2\sin 2x\sin x\Rightarrow 2\sin x(\cos 2x+\sin 2x)=0\Rightarrow\begin{matrix}2\sin x=0\\ \cos 2x+\sin 2x=0\end{matrix}$$

$$\begin{cases}\sin x=0\Rightarrow x=0^\circ;\quad x=180^\circ\\ \frac{\sin 2x}{\cos 2x}=\frac{-\cos 2x}{\cos 2x}\Rightarrow\tan 2x=-1\Rightarrow\begin{matrix}2x=135^\circ\Rightarrow x=67{,}5^\circ\\ 2x=315^\circ\Rightarrow x=157{,}5^\circ\end{matrix}\end{cases}$$

35.- a) $\sin 3x=\sin 2x\cos x+\cos 2x\sin x=2\sin x\cos x\cos x+(\cos^2 x-\sin^2 x)\sin x=$
$2\sin x\cos^2 x+\sin x\cos^2 x-\sin^3 x=3\sin x\cos^2 x-\sin^3 x$

b) $\sin 3x - 2\sin x = 0 \Rightarrow 3\sin x\cos^2 x - \sin^3 x - 2\sin x = 0 \Rightarrow$

$$\sin x(3\cos^2 x - \sin^2 x - 2) = 0 \Rightarrow \begin{cases} \sin x = 0 \Rightarrow x = 0^\circ; \quad x = 180^\circ \\ 3\cos^2 x - 1 + \cos^2 x - 2 = 0 \Rightarrow 4\cos^2 x = 3 \end{cases}$$

$$\cos^2 x = 3/4 \Rightarrow \begin{matrix} \cos x = \sqrt{3}/2 \Rightarrow x = 30^\circ; \quad x = 330^\circ \\ \cos x = -\sqrt{3}/2 \Rightarrow x = 150^\circ; \quad x = 210 \end{matrix}$$

37.- $\sin\alpha\cos 2\alpha - \cos\alpha\sin 2\alpha = \sin\alpha(\cos^2\alpha - \sin^2\alpha) - \cos\alpha(2\sin\alpha\cos\alpha) =$

$\sin\alpha\cos^2\alpha - \sin^3\alpha - 2\sin\alpha\cos^2\alpha = -\sin^3\alpha - \sin\alpha\cos^2\alpha =$

$-\sin\alpha(\sin^2\alpha + \cos^2\alpha) = -\sin\alpha\cdot 1 = -\sin\alpha$

38.- a) $\left.\begin{matrix} x + y = 120 \\ \sin x - \sin y = 1/2 \end{matrix}\right\} \Rightarrow \begin{matrix} y = 120 - x \\ \sin x - \sin(120 - x) = 1/2 \Rightarrow \end{matrix}$

$$\sin x - \sin 120\cos x + \cos 120\sin x = 1/2 \Rightarrow \sin x - \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = 1/2 \Rightarrow$$

$2\sin x - \sqrt{3}\cos x - \sin x = 1 \Rightarrow \sin x - \cos\sqrt{3}\cos x - 1 = 0;$ Haciendo $\cos x = \sqrt{1 - \sin^2 x}$ tenemos:

$$\sin x + \sqrt{3}\sqrt{1 - \sin^2 x} - 1 = 0 \Rightarrow \left(\sqrt{3(1 - \sin^2 x)}\right)^2 = (1 - \sin x)^2 \Rightarrow 3(1 - \sin^2 x) = 1 + \sin^2 x - 2\sin x \Rightarrow$$

$$3\sin^2 x - 2\sin x - 2 = 0 \Rightarrow 2\sin^2 x - \sin x - 1 = 0 \Rightarrow \begin{matrix} \sin x = 1 \Rightarrow x = 90^\circ; \quad y = 30^\circ \text{ Es válida} \\ \sin x = -1/2 \Rightarrow \begin{matrix} x = 330^\circ \\ y = 210^\circ \end{matrix} \Rightarrow \text{No están en el primer cuadrante} \end{matrix}$$

b) $\left.\begin{matrix} \sin^2 x + \cos^2 y = 1 \\ \cos^2 x - \sin^2 y = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sin^2 x + \cos^2 y = 1 \\ 1 - \sin^2 x - 1 + \cos^2 y = 1 \end{matrix}\right\} \begin{matrix} \text{sumando} \\ \text{ambas} \end{matrix} \Rightarrow 2\cos^2 y = 2 \Rightarrow \cos^2 y = 1$

$$\cos^2 y = 1 \Rightarrow \cos y = \pm 1;\ \sin^2 x = 0;\ \begin{matrix} \sin x = 0 \\ \cos y = 1 \end{matrix} \Rightarrow \begin{matrix} x = 0^\circ \\ y = 0^\circ \end{matrix};\ \begin{matrix} \sin x = 0 \\ \cos y = -1 \end{matrix} \Rightarrow \begin{matrix} x = 0 \\ y = 180^\circ \end{matrix} \begin{matrix} \text{No válido} \\ (2^\circ \text{ cuadrante}) \end{matrix}$$

c) $\left.\begin{matrix} \sin x + \cos y = 1 \\ x + y = 90 \end{matrix}\right\} \Rightarrow \begin{matrix} y = 90 - x \\ \sin x + \cos(90 - x) = 1 \Rightarrow \sin x + \cos 90\cos x + \sin 90\sin x = 1 \Rightarrow \end{matrix}$

$$\sin x + \sin x = 1 \Rightarrow 2\sin x = 1 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = 30^\circ; \quad y = 60^\circ$$